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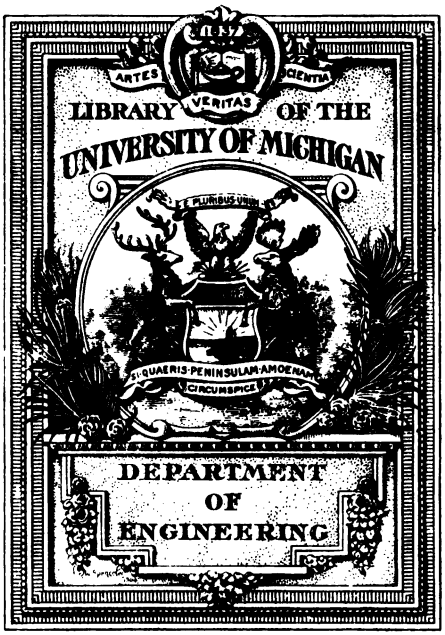
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MATERIALS AND CONSTRUCTION

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MATERIALS AND CONSTRUCTION

A

THEORETICAL AND PRACTICAL TREATISE

ON THE

STRAINS, DESIGNING, AND ERECTION OF WORKS OF CONSTRUCTION

By FRANCIS CAMPIN, C.E.

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PREFACE.

In writing the present treatise, my object has been to produce a work dealing comprehensively with the subject of materials and their use in certain branches of constructive art, viz. the massive works usually intrusted to civil engineers and architects, and throughout I have carefully avoided the introduction of the higher branches of mathematical investigation; and in so doing I have not omitted problems of the classes usually treated by high mathematical processes, but have substituted simpler, but equally convincing, lines of argument for the more abstruse processes of analysis.

It will be found that algebraical arithmetic, or simple forms of equations, supply the basis of calculation, and this basis is indeed amply sufficient for all the theoretical reasoning that is called for in the consideration of the practical problems engaging our attention.

From the above remarks it will be seen that the work is designed especially for all those readers who desire to become thoroughly acquainted with the theories of structures and the practical application of results in the simplest way, and not as a mathematical exercise. It may be advisable to say a few words in explanation of the stress I lay upon the importance of this simplicity of calculation.

There are comparatively few of those entering upon a mechanical profession who are thoroughly accomplished mathematicians, and once launched upon the business of life—or what is equivalent to it, the probationary term which precedes actual remunerative employment—the tyro will not desire to give time to the study of abstruse science, beyond the point where it ceases to be absolutely necessary for his purposes; and there are many who have only learned these exact sciences bit by bit as they have found them necessary.

Another matter of common consideration is, that even in those who have become proficient at school and college in pure mathematics, this knowledge, unless sedulously maintained and reinforced by after-study, rapidly decays, and is often only with great difficulty revived; and the time absorbed by this reinforcement or revival is generally required for the purposes of more directly practical study.

Although all structures should combine in themselves both strength and stability, I have, for the sake of clearness, separated the two classes as far as can conveniently be done for theoretical investigation; showing, however, their necessary connection in suitable places.

In the examples taken to illustrate the methods of calculation, I have carefully selected cases such as occur in every-day practice, and carried them through, in order to leave no doubt or difficulty as to the practical application of the formulæ.

In conclusion, I would add that this work is not intended in any way as an elementary introduction only to the science of construction, but deals fully and finally with all the subjects included in its syllabus.

FRANCIS CAMPIN.

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MATERIALS AND CONSTRUCTION.

CHAPTER I.

INTRODUCTORY — CONSTITUTION OF MATTER — ELASTICITY—INTERNAL FORCES—EXTERNAL FORCES.

THE adaptation of the various materials furnished by nature, or elaborated from natural products, to the purposes of constructive art, necessarily requires a certain amount of practical and technical knowledge on the part of the constructor; but it should not be imagined that this knowledge is of so special a character, or so intricate in its details, as to limit its attainment to a select few, and it will be shown to be based merely upon careful and persistent observation, whence are derived data upon which, by an ordinary course of reasoning, the technical principles are founded.

In the present treatise I shall carefully avoid clothing the demonstrations and investigations which will occupy our attention in the language of high mathematics, and shall use such expressions as may be familiar to those who have not been specially educated to consider the subjects here dealt with.

It is obvious that the commencement must be made by an inquiry into the nature of the materials presenting themselves to our notice, and therefore we must consider the constitution of solid matter generally, in order to

ascertain by what properties it is rendered available for constructive purposes, and how these properties act in enabling it to resist external agencies tending to its destruction or deterioration. In using the term destruction, I limit its application to the destruction of a certain body, only in so far as it is rendered useless for some specific purpose to which it is applied.

All so-called solid matter really consists of numerous aggregations of very small particles, each such aggregation forming a molecule, and these molecules are of the same materials as the whole mass. If by mechanical means a solid mass be broken down into small particles, each particle is still of the same nature as the mass of which it formed a part—that is, its chemical composition is not affected.

In the solid mass itself the molecules are not in actual contact with one another; the mass is not really *solid*, but is full of pores, or interstices; so the particles are sustained at some distance from each other, this distance forming a characteristic of the material—thus lead is a close and cork an open material.

That the molecules of matter are not in contact is evident from the contraction of bodies under reductions of temperature, or from the effects of externally acting mechanical forces.

If a mass of matter at rest be acted upon in such a way as to distort its shape, and when the acting force is removed it resumes that shape, such a body is said to be "elastic," and it is by virtue of its elasticity that it recovers its normal form. If the original shape is exactly resumed, then the elasticity of the material is said to be perfect, and no disarrangement of its constituent molecules has occurred; but if the elasticity is impaired, then a permanent set or distortion has taken place, which may or may not alter the actual strength of the material according

to the circumstances under which it has been brought about.

Let us picture to ourselves the condition of the small particles, or molecules, as they exist when aggregated together in a mass of matter at rest. All the particles are standing apart in space, balanced at certain distances from each other by forces which must be of antagonistic natures, such as attraction and repulsion; for, did attractive forces alone act, the molecules would be in contact, while under the sole influence of repellent agencies they would fly asunder and the mass assume the form of highly attenuated gas, expanding and spreading without limit.

It is not my purpose to enter upon any inquiry as to the nature or quality of these forces or their origin; it is sufficient for the present object to know from observation that they exist in various degrees of intensity in all the materials used in construction, and that we can ascertain their relations to external force by experiment, and so arrive at data as to the inherent strengths of different kinds of material.

When a body is at rest, the two forces must just balance each other, and so keep the molecules in equilibrium. These forces are called the Internal Forces.

By the action of any external force upon a body the equilibrium of its molecules is disturbed; thus, if pressure be brought to bear upon it simply, the internal attractive force is assisted in the direction of the pressure, and the particles approach each other in that direction, partially overcoming the repelling molecular forces, although these again may act laterally, causing the body as it shortens to become wider: on the other hand, a pulling force will aid the repellent forces and lengthen the body, but those repellent forces become weakened in their lateral action, and as the body stretches it becomes narrower. These are direct forces, producing stress upon material, and into

them all other forces must be reduced in order to compare them with the direct resistances of solids to distortion or fracture.

It was discovered a long time ago that the amount of extension or compression a body undergoes—that is, its actual lengthening or shortening—is, if its elasticity be perfect, in direct ratio to the intensity of the force producing it; thus, if 10 tons will lengthen a certain piece of iron by one ten-thousandth part of its original length, then 20 tons will lengthen it by one five-thousandth part, and so on.

For purposes of comparison and calculation it is necessary to have tabular numbers to give the elasticity of the various materials; for this end certain co-efficients, as they are called, have been thus determined:—Let the weight required to produce a certain measurable elongation of any given material be determined, the body experimented upon being of equal dimensions throughout its length, one inch square and perfectly straight, with the weight so adjusted as to act accurately in the direction of its length. Then multiply the weight by the *original length* of the bar, and divide the product by the extension of the bar under the influence of the weight. The quotient is called the "modulus of elasticity" of the material, and is the weight that would, were such a thing possible, stretch the bar to twice its original length.

The "modulus of elasticity" having been once determined, it is easy to calculate the extension or compression of a body of given length and sectional area under any given stress, as we have only to multiply the length by the strain per sectional square inch, and divide the product by the modulus of elasticity; as, acting through short distances, the elastic resistance to compression is measured by the same modulus as the resistance to extension.

I will now consider generally the action of external

forces that do not lie directly in the line of the internal resisting forces, for it will presently be found that it is with this class of strains we shall principally have to deal in regard to structures of all descriptions.

In Fig. 1 let A B C D be a side view of the layers of molecules forming a beam supported at the points C D; the material being supposed to be unstrained, the mole-

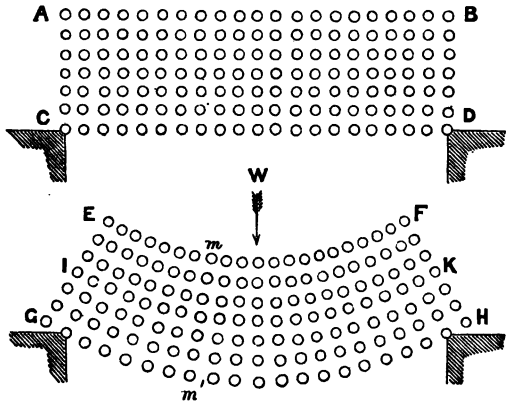

Fig. 1.

cules will be uniformly arranged as indicated by the dots. Now let a force, W, be brought upon it so that it becomes bent, then it will assume the form shown by the diagram E F G H, where G and H are the points of support, which react upwards with a total force equal to W acting downwards. It is evident that the molecules in the upper layer, E F, are crowded together by the change of form, whilst those in the layer G H are stretched apart; the layers next to these external ones undergoing similar changes in a less degree, until at the layer I K there is neither crowding nor stretching. There will be compressive strain on the concave side then, of the beam and

tensile strain on the convex side, and the beam will endeavour to regain its normal form by the repulsion of its molecules, *m*, on each other, and the attractions of those, *m'*, tending to restore each vertical layer, *m m'*, to its original vertical position, by revolving it about a point in the layer I K. I K is termed the neutral layer, or neutral axis.

This illustrates the internal resistances of a beam to bending as a number of pushing and pulling efforts acting along the two arms of a lever, those of opposite kinds being on opposite sides of the fulcrum.

The intensities of these elastic resistances and their effects will be exactly ascertained when treating of the behaviour of materials subjected to bending or transverse stress.

When material is placed between two edges so that on their being pressed towards each other a tendency to sever it arises, the strain is known as shearing strain; this occurs on the rivets joining plates on which longitudinal forces act: a diagram will clearly illustrate the manner in which this stress acts.

In Fig. 2 let A and B be two edges acting upon the opposite surfaces of the body E F in such a manner as to shear it along the line *c d*. The action tends to force each molecule downwards, and away from that opposite it, and the result will be different according to the nature of the material. It may happen that as a layer of molecules is forced down from the opposite layer, in coming opposite the layer next below, it will be attracted by that with sufficient force to maintain the continuity of the mass, if the shearing force be now stopped. Such a case is ex-

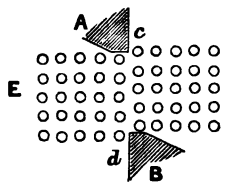

Fig. 2.

hibited by lead; but if this does not occur, the mass will be divided at *c d*, and the molecules having been forced asunder, it is evidently tensile stress that has been called into action; therefore, we should expect the shearing resistance of a body to be for equal areæ equal to its tensile resistance, and this is found to be the general rule in non-crystalline matter. The failure of bodies under compressive strain occurs in several different ways. If the member acted upon is not absolutely straight and of equal resistance per square inch in every part of any given section, it will give way by bending or crippling, without being actually crushed; and when it so happens the convex side will be in tension and the concave in compression, the same as in a beam under transverse load. As it is commercially impossible to secure homogeneity or uniformity of substance throughout the materials we use, it follows that failures under compression must, in elements of any length compared with their breadth or thickness, occur in the manner just described.

In dealing with this question we are at the outset met by the difficulty that we do not quite know how the work will break, for it may be by transverse bending, or it may be by a process of crushing or splitting, and this will depend not only on the nature of the material, but also on the circumstances in which it is placed. It is very seldom that an actual flattening out will occur practically.

In Fig. 3 the methods of crushing are shown at A; the upper part has wedged asunder the lower, and here there evidently are in action both shearing force along the faces of division, and tensile stress on the parts forced open. In fracture, as shown at B, shearing stress alone seems to come into play, but the angle will depend upon the qualities of the material experimented on. The calcula-

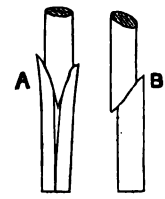

Fig. 3.

tions used in determining the proportions of columns and elements subject to compressive stress are of an empirical nature, being derived from extensive series of experiments.

It is impossible to examine the internal action of the external forces without being struck by the manner in which every kind of force seems to incline towards conversion into tensile stress on the molecules, and it must always happen that rupture ultimately occurs by these molecules being pulled or driven beyond the limits of their spheres of mutual attraction, for so long as they remain within those spheres they cannot be separated and fall asunder. When bodies are distorted, and so kept for a length of time, there is observed a tendency of the molecules to rearrangement by equalising the internal strains, and the less perfect the elasticity of the material the more extensive will be the adjustment thus occurring.

Passing on beyond the limit of elasticity of a material, we may yet stop short of actual rupture, but the integrity of the substance will have been invaded, some *permanent* distortion will have been caused, and consequently we may assume that a proportionate amount of damage has been done.

I would here warn my readers against the abuse of the term "permanent set," as it is applied commonly to effects which have no more similarity to that which it really means than has the lightest stress to absolute fracture of material.

Permanent set, in reference to structural details, means, actual molecular alteration in the internal arrangement of the substances acted upon, and it is incorrect to apply it to any other effect. The most common misapplication is as applied to the permanent subsidence of works due to the joints falling into their bearings, and defects in the mode of erection; but some folks have even gone to the extreme of applying the term to the deflection due to a permanent load, which really should be called the permanent deflec-

tion. I have thought it desirable to allude to this matter, as nothing tends more to confusion than the slovenly misapplication of technical terms.

It is evident that the limit of elasticity cannot be exceeded with impunity, and practically a sufficiently wide margin of strength must be left. It seems only reasonable that the *working strength* of material should be taken in some proportion to the *elastic limit of resistance* and not to the *ultimate resistance* of the material, but up to the present time this latter ratio has been the guide almost universally.

In some substances, however, it must be remarked the range of elasticity is so extremely small as not to be observable, as in some kinds of stone, brick, &c., and the material appears to give way without previous alteration of shape, although we know that some such alteration must take place before the normal arrangement of the internal forces can be altered.

As to the factors of working strength to be used in practice, I shall give those under the special headings of the various descriptions of constructive details upon which they bear.

The resistances offered by structures to disturbing forces may be put in two classes:—1st, the resistance due to the strength of the material—that is, to the cohesion of its constituent molecules; this is properly called the strength of the work.

2nd. The resistance offered by the dead weight of material, which may operate in opposition to an overturning effort, or to a force tending to slide its mass bodily on the surface on which it stands, or to both combined; this resistance is the "stability" of the work.

In the first form the force is directed to overcome gravity by causing a body to lift and revolve about one of its edges until the centre of gravity falls without such edge, when the mass will altogether upset; in the second, the

resistance to be overcome is that of the friction of the mass on the surface upon which it rests.

I will here point out the nature of the resisting force of friction between surfaces. The surfaces are taken to be physically smooth and free from viscosity or any special property of attraction for, or repulsion of, each other.

On account of the elasticity of matter, it follows that if a body rest upon another of larger surface than its surface of contact, it will to some extent sink into it, compressing the parts immediately beneath; hence, if the upper body be pushed along, it must as it were be pushed into a *higher part*, which, in its turn, sinks under the weight imposed upon it, so that virtually, in moving the upper body, we are constantly pushing it uphill. As time is required for a substance to be compressed, it is evident that if the upper body be moved rapidly upon the lower it will not at any time sink as much as if it were allowed to rest on one spot; hence the friction at starting, which is the friction of rest, is greater than the friction of motion; and the friction of slow motion will be greater than that of rapid motion, so far as the surfaces themselves are concerned.

The friction of surfaces, when the pressures upon them are slight, compared to what would be required to injure them, are taken as certain fractions of the insistent pressures, regardless of the areas in contact; but it is evident that regarding the matter strictly, there should be some variation with area of surface exposed to a varying pressure: this, however, does not seem to be of sufficient magnitude to demand practical consideration.

Having determined the properties and capabilities of the materials at our disposal, it follows to consider the best modes of applying them for economic purposes: to arrive at these, the nature of the forces presenting themselves as acting upon structures and machinery must be carefully

ascertained, in order that means may be taken for so controlling their directions and modifying their intensities, that our materials may be brought in the most advantageous way to oppose them.

We have, then, this general problem to be divided and specially applied in each of the numerous cases that arise in every-day life. There are two numerical values or sets of quantities, which must be made equal to each other; they are the action and reaction of mechanical equilibrium, which can only be obtained by the opposing of opposite and equal forces, or of a number of forces that may be divided and resolved into two opposite equal forces. In the question before us, one set of forces is that due to loads, pressure of wind, and other external forces; and the other consists of the resisting internal forces of the structure, or of its gravitative effort, or its resistance to sliding, or the sum of any of these separate modes of resistance.

CHAPTER II.

GENERAL PROBLEMS—DIRECTION OF A FORCE—MOMENT OF FORCE—EQUILIBRIUM OF FORCES—SUBSTITUTION OF FORCES.

THE direction of a force is a straight line: we cannot speak of a force as moving in a curve, and the direction of a force at any moment is the direction in which it is then acting.

If a force in action does not exert itself directly upon some body, but acts about some point as a fulcrum, then the intensity of the force multiplied by the distance at which it acts from the point about which it acts is called the "moment" of such force. Any form of simple lever exhibits two of these moments acting in opposition to each other, and by employing the method of calculating by moments all cases of forces acting about a centre may be dealt with. The distance at which the force acts from the point is the least distance from that point to the direction of the force under consideration. In Fig. 4, let the straight line *a b* represent the direction of a force P, and let it be acting about a point at *c*. If necessary, produce the line of direction *a b* to *d*, and from *c* draw the straight line *c e* at right angles to *a b d*: then *c e* will be the distance at which the force P acts about the centre *c*, and the moment of such force will be P×*c e*. If P=4 tons and *e e*=8 ft., the moment of

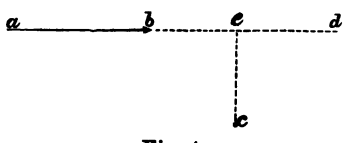

Fig. 4.

force would be called 32 ft.—tons. We see from this that one moment may in value represent many different combinations of force and leverage or distance, and before going further it may be best to show the relations of equal moments of force.

A force at rest, or as it is technically called static force, signifies a pressure, and possesses only intensity or degree: if, however, the force is exercised through a space, it becomes dynamic; then mechanical work results. If the equilibrium of any mechanical system is disturbed, there must be some kind of motion before it is restored, even if this movement be no more than the molecular yielding of the material. The circumferences of circles are in direct ratio to their diameters, therefore circumferal boundaries—arcs—of any proportionate parts will vary as the diameters. In Fig. 5 let *a b* be a bar carried upon a centre *c*, on which it is capable of revolving; at the end *a* let a force P act in one direction, and at the end *b* a force W in the other. In one revolution of the bar the point *a* will pass through a distance, which is to that passed through by the point *b* as the length *a c* is to the length *b c*; hence if the work done at both points be equal, the product of the force P into the distance it acts through must be equal to the product of W into its travel, and the forces must be in inverse ratio to the circles they describe, and therefore to the radii *a c*, *b c*. If *a c*=2*b c*, then must W=2 P, in order that the work done in passing through a given angular distance shall be the same on both sides of the fulcrum or centre *c*.

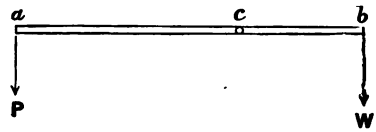

Fig. 5.

If, then, any moment be known, and one of the terms of a moment equal to it be given, then the other term can be found. Let the above system be in equilibrium, *a c*=13 ft.,

$b\ c$=3 ft., W=7 tons, required to find P. The moment of W is W$\times b\ c$=7$\times$3=21 ft. tons; but $a\ c$=13 ft., hence P=21 ft. tons$\div$13 ft.=1·61 &c. tons; for if

$$W \times b\ c = P \times a\ c \therefore P = W \times b\ c \div a\ c.$$

I will now consider the action of inclined forces conjointly at a point. In Fig. 6 let the two forces P P′ converge upon a point a. Produce the directions of these forces indefinitely, and mark off $a\ b$ to any convenient scale to represent the force P′, and $a\ c$ to represent P from b and c; draw $b\ d$ parallel to $a\ c$ and $c\ d$ parallel to $a\ b$, meeting at d; join $a\ d$. Then $a\ d$ will represent a force equal in its effects to the combined forces P, P′. For suppose the forces to be expended in imparting moment, let P′ impart to a body at a a velocity that will carry it to b in one second, then let P impart a velocity that will carry it to d ($b\ d$ is equal to $a\ c$) in one second, it will be at the same point as if acted on by a force represented in amount and direction by the straight line $a\ d$, and if the two forces P, P′ act together instead of consecutively, the body will travel along the line $a\ d$ in one second. In working problems by this, the parallelogram of forces, the forces are laid down to scale on the diagram; as for instance putting 50 tons to an inch, the same as in making maps we may put 100 ft. equal an inch. Some students are a little puzzled at first about scaling down forces on diagrams, but a moment's consideration will show that the matter is quite simple. These two methods of treating the relations of forces to each other may be applied as a mutual test of accuracy, and an example scaled out will here prove their agreement, and also serve to show the method of applying them. On com-

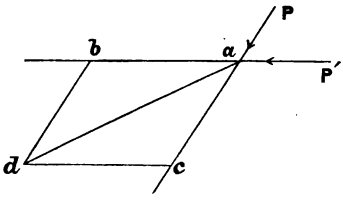

Fig. 6.

mencing to work graphically the student should provide himself with an accurately divided scale, and for this purpose that known as a diagonal scale is very convenient, as it admits of the lengths being easily taken off by the compasses or dividers, without trying the sight by the proximity of the division lines. In the example Fig. 7, I have taken for my scale 20 tons to one inch. P and P′ represent 12 and 18 tons respectively; marking these off and scaling the diagonal, it is found to measure 29·175 tons, which with its direction should equipoise the forces P, P′. Take any convenient point *a* as a centre for the forces to act about, then the moment of the force P″ should be equal to the

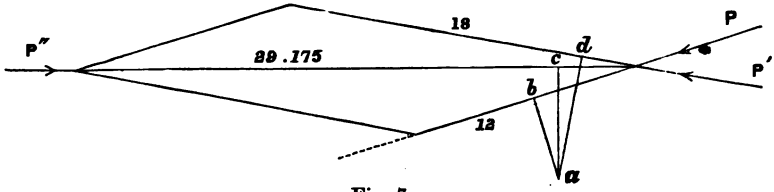

Fig. 7.

sum of the moments of the forces P, P′ about *a*. The distances at which the forces act will be found from the perpendicular *a b*, *a c*, *a d*, which are found to measure (they may be taken on any scale, so long as the same scale is used for all three) respectively *a b*=355, *a c*=406, *a d*=420, $355\times12+420\times18=4260+7560=11820$, which is the sum of the moments of the forces P, P′ about the point *a*. $29{\cdot}175\times406=11845$, which is the moment of P″ about the point *a*. The slight discrepancy is due to errors of measurement, and it will be observed that even on the very small scale to which this diagram is drawn, those errors are practically nothing, being about $\frac{1}{2}$th of one per cent. Of course the larger the scale adopted, the greater will be the accuracy of the results.

I now come to what may be called the process of substitution of forces: we have certain natural forces of which the directions are known, and for these we have to substitute equivalent forces going in some other directions, in which they may be met by the resistances of the materials used in the structure opposed to them.

Setting aside for the present lateral wind pressures and the like, the natural forces arising to be dealt with will proceed from gravitation, and hence will primarily be vertical in direction. The fact of these forces acting vertically affords great convenience for the description of the directions of parts of structures relatively to the original forces, as by the amount of elevation corresponding to a given horizontal extension: thus if two points be horizontally 3 feet apart, but one is 2 feet higher than the other, the inclination will be 2 to 3. These quantities being known, the distance between the points, and therefore the length of the element joining them, may be calculated from the property of a right-angled triangle, which is that in any right-angled triangle the sum of the squares of the sides enclosing the right angle is equal to the square of the remaining side.

It will readily be seen that if a structure be composed of bars jointed together so as to form a rigid combination, and then force be applied to it tending to distort it, such force will be split up into others, which will pass along and be resisted by the elastic strength of the component bars of the frame. If in Fig. 8, *a b*, *a c* represent two bars attached at *b c* to a solid mass, and a weight W is hung on at the joint *a*, the force due to the gravitation of this weight will be

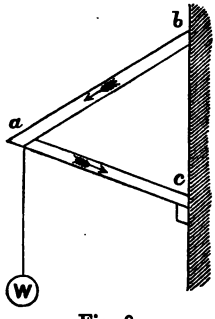

Fig. 8.

split up into two others, one tending to pull *a b* away from and the other tending to thrust *a c* into the sustaining mass. The relations of the intensities of these strains to the weight W may be determined by the parallelogram of forces, and so the materials proportioned to sustain them. If the bars *a b*, *a c*, instead of being attached to a solid mass as shown, were connected to another framework of bars, there would be a further splitting up, or resolution of strains to be determined in a similar manner, and so we may go on, commencing with the primary load, and trace the resulting strains through all the bars of the most complicated structures up to the point where the strains pass away into the earth. By drawing the skeleton of centre lines of a framed structure, the diagram of strains will be formed, and the relations of the strains will be seen to vary with the relations of length of the various elements; but to this subject I shall return to deal particularly with the application of the methods to practical cases.

Having now acquired a clear conception of the manner in which forces internal and external come into action and produce definite results, the way is clear for the application of this knowledge to special cases, and for the elaboration of a series of definite formulæ or rules for the ordinary requirements of constructive art, and in the first place I shall treat of structures which depend upon the elastic strength of the materials composing them for their security and permanence.

CHAPTER III.

BENDING STRESS.

As the centres of gravity of surfaces and bodies will now constantly be referred to, a few words on the subject now may facilitate the subsequent operations.

The centre of gravity of a surface or solid is that point on which it balances exactly, and is the point about which the moments of weight of all the molecules in one direction will equal the moments of weight of all the molecules in the opposite direction. There can evidently be only *one* such point in any body or surface. A surface may, however, be balanced upon a knife-edge in more directions than one, but all the lines of direction thus found will intersect each other at the centre of gravity of the surface. The centre of gravity is the point at which we may consider the whole weight or effect of the surface or body as concentrated; and if any figure is freely suspended from a point, the centre of gravity will hang *vertically under such point.* This fact furnishes a simple method by which the centre of gravity of any figure, however irregular, may be ascertained. Let it be required to find the centre of gravity of an irregular figure, *a b c*, Fig. 9. Cut the figure out in paper or Bristol board; pierce holes at two corners, as at *a* and *c*. Suspend the figure on a pin stuck in a

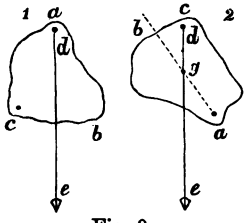

Fig. 9.

vertical board, and on the same pin hang a plumb line *d e*, and when it has become steady mark its direction and draw it on the figure, then the centre of gravity must be *somewhere* in the line thus drawn. Now suspend (as at 2) the figure by the hole at *c*, again apply the plumb line *d e*, and where it intersects the line already drawn will be the centre of gravity *g*.

There are many, most in fact, of the forms with which we shall have to deal, of which the centre of gravity may readily be found without having recourse to this tentative method. Take a triangle, *a b c*, Fig. 10, for instance; this triangle may be regarded as being made up of indefinitely narrow strips parallel to *b c*, then each strip would balance on its centre; so if the side *b c* be bisected in *e* and *a e* joined, *a e* is a line passing through the centres of gravity of all the parts, and therefore through that of the whole system. Similarly, by bisecting *a b* in *d* and joining *c d*, another line is found passing through the centre of gravity of the system; hence the intersection of these two lines determines the centre of gravity *g* of the triangle.

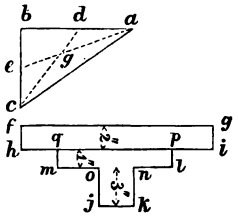

Fig. 10.

If a surface, then, is of symmetrical form about a rectilinear axis, its centre of gravity is somewhere in such axis, and if there be two axes of symmetry the centre of gravity will be at their intersection.

The centre of gravity of an area such as *f g k j* may be determined by finding the sum of the moments of the separate constituent areas about any convenient axis, and dividing it by the sum of the areæ, the quotient being the distance of the centre of gravity of the system from the axis chosen.

Let the dimensions of the figure be $fg=15$ inches, $lm=9$ inches, $jk=2{\cdot}5$ inches, the other dimensions being as figured, and the axis selected about which to determine the moments be the boundary fg. The centre of gravity of each symmetrical element being its physical centre.

Then the moment of the part—

$fgik$ = area × distance from axis $=(15\times2)\times1 = 30{\cdot}00$

$lmqp$ = ,, × ,, ,, $= (9\times1)\times2\frac{1}{2} = 22{\cdot}50$

$nojk$ = ,, × ,, ,, $=(2{\cdot}5\times3)\times4\frac{1}{2}\times33{\cdot}75$

$86{\cdot}25$

$$\frac{86{\cdot}25}{(15\times2)+(9\times1)+(2{\cdot}5\times3)}=\frac{86{\cdot}25}{46{\cdot}5}=1{\cdot}855\text{ inches}=\text{distance}$$

of centre of gravity of the whole figure from the axis fg.

This term, centre of gravity, of course applies strictly to the centre of action of the force of gravitation, but custom has caused it to be used in speaking of the centres of forces other than that of gravity, to which its position applies, such as the centre of gravity of a resisting area of material; thus we say the centre of gravity of the flange of a girder, when that point is referred to in relation to the tensile or compressive stresses on the area of which it is centre of gravity.

The weights or loads producing strain are generally of geometrical form, having a centre of figure which, if the bodies be of uniform or homogeneous constitution, is also the centre of gravity; and if the load be not of such a character, it must be, for purposes of calculation, divided up into smaller parts, of which the centres of gravity may be observed.

I should advise the student to take for practice at haphazard a number of figures, and having determined the centres of gravity by calculation, to check them by the tentative method, in order to give him facility in working and confidence in his results.

In Fig. 11 let A B represent a piece of a rectangular beam at rest, and let $a\,a'$, $b\,b'$ be the edges of two imaginary planes intersecting it at right angles to its length. Now by some external force let it be bent as shown at C D, then will the planes $a\,a'$, $b\,b'$, which before were parallel, become inclined to each other, the ends $a\,b$ receding and the opposite ends $a'\,b'$ approaching each other; there will, therefore, be some intermediate position where the distance between the planes $a\,a'$, $b\,b'$ will remain the same as it was prior to the beam being bent. Let this position be indicated by the line o', o, o''; it is called the *neutral axis* because it is unaffected by the external forces bearing upon the beam, and comprises an indefinitely thin layer of material running through the beam longitudinally, and at right angles to the plane in which the beam is curved.

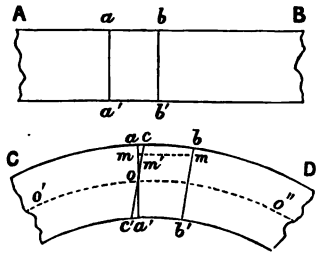

Fig. 11.

At the point o, where the neutral axis intersects the plane $a\,a'$, draw $o\,c'$ parallel to $b\,b'$; then $a\,c$ will represent the elongation of the fibre $a\,b$, and $a'\,c'$ will similarly represent the shortening of the fibre $a'\,b'$, and from these two positions the elongation and shortening of the fibres, or laminæ, diminish to the point o, where neither shortening nor lengthening occurs.

Let $m\,m$ be a lamina at any place distant x from the neutral axis, then its extension $m\,m'$ will be in direct ratio to the value of x. $\frac{m\,m'}{m\,m} \times$ modulus of elasticity will be the strain per sectional square inch upon the lamina $m\,m$; and the strain being known on any given lamina, that on any other of which the distance from the neutral axis is known

can be found by proportion, for if $m\,m$ be distant x, and $n\,n$ distant x' from the neutral axis,

$$\frac{m\,m'}{m\,m}\times \mathrm{E} : \frac{n\,n'}{n\,n}\times \mathrm{E} :: x : x'$$

where E=modulus of elasticity.

The value of $\frac{m\,m'}{m\,m}$ is determined arbitrarily, or rather, I should say, its maximum value is limited by the factor of strain permitted to be used. If we were experimenting upon the actual breaking weight of the material, its ultimate strength would be this limit, but in practice it is measured by the working strain.

The various laminæ are now resisting by their elastic forces the distortion to which the beam is subject, the tension or thrust of each lamina acting about the point o in the neutral axis, as about a fulcrum, with a tendency to restore the normal straight form A B.

Let b=breadth of the beam in inches, d=its depth in inches, s=maximum strain in tons per sectional square inch, z=a divisor indefinitely great in numerical value.

We cannot conceive a lamina of material that is a mathematical plane—has *no thickness*, in fact; and if in our beam the lamina have any thickness, the one surface will experience a greater strain than the other; the approximate moment of resistance of any lamina will be represented by its breadth multiplied by its thickness, taken as an indefinitely small fraction of the depth of the beam, by the resistance per square inch, and by its distance x from the neutral axis. Let M′=this moment, then

$$\mathrm{M}' = b\times\frac{d}{z}\times s\times x.$$

But as we cannot assign a value to z, we must endeavour to find an expression for the sum of the moments of resistance of all the separate laminæ.

As in the triangle, $a\,o\,c$, $a\,c$ and the lines parallel to it

represent the direct strains on and elastic resistances of the individual laminæ; and as the sum of all these lines makes up the triangular area *a o c*, that area may be regarded as representing the sum of all these different elastic resistances, and their moment will be this sum multiplied by the mean distance—that is, the mean physical distance, or distance of the centre of these forces—from the neutral axis of the beam.

The centre of the forces will naturally be the centre of gravity of the triangle, as the area of that figure represents both the amount and position of the forces. If a triangle be drawn, and the centre of gravity found by bisecting two sides and proceeding as detailed on a previous page, it will be found that the centre of gravity on a line drawn from the centre of its base to its apex is at one-third of its length from the base; this, then, will be two-thirds of its length from the neutral axis, and it is at this point that all the forces on one side of the neutral axis may be regarded as concentrated. Of course there will be a similar concentration of forces at the centre of gravity of the triangle on the opposite side of the neutral axis, and the moment of the forces on one side of the neutral axis will in this case be equal to the moment of the forces acting on the other side of the neutral axis. It is important to bear in mind the necessity of the equality of forces on both sides of the neutral axis of the beam, for it is by this equality that the position of the neutral axis is determined.

Putting these quantities into symbolical form, we have, calling M' the moment of resistance of the section on one side of the neutral axis, and M the total moment of resistance of the section, and replacing *a c* in the area of the triangle by its value *s*,

$$M' = b \times s \times \frac{d}{4} \times \frac{2}{3}\left(\frac{d}{2}\right) = \frac{s.\,b.\,d^2}{12}. \quad M = 2\,M'' = \frac{s.\,b.\,d^2}{6}$$

in which $\frac{s.d.}{4}$=the area of the triangle $a\,o\,o$, and $\frac{2}{3}\left(\frac{d}{2}\right)$= the distance of its centre of gravity from the neutral axis.

It will, perhaps, render the meaning of this expression more distinct if I take an example and give numerical values to the different terms.

Let the breadth of the beam section be 11 inches, its depth 8 inches, and its resistance per sectional square inch in both tension and compression 4 tons, then its moment of resistance when under that strain will be—

$$M=\frac{s\times b\times d^2}{12}=\frac{4\times 11\times 8\times 8}{12}=234\cdot\dot{6}\text{ inch tons.}$$

So that, assuming the beam to be without its own weight, it would at this strain support at its free end 10 tons, if it were fastened by one end into a solid wall and projected $23\cdot4\dot{6}$ inches from that wall, because then the moment of strain would be $23\cdot4\dot{6}\times 10=234\cdot\dot{6}$ inch tons, equal to the above moment of resistance.

If the section is symmetrical the neutral axis is in the centre of gravity, when the elastic resistance per square inch is the same in tension as compression, otherwise the neutral axis will occupy some other position; and if the section be not symmetrical, the neutral axis must be found from the equality of moments of resistance.

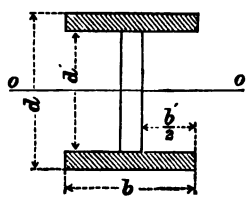

Fig. 12.

If the beam is symmetrical, but not solid, being of some section like that shown in Fig. 12, the moment of resistance will be found by first taking that of the whole area enclosing the section, and then deducting the moments of resistance of those parts that are away, thus:—

$$M=\frac{s.\,b.\,d^2}{6}-\frac{s'.\,b'.\,d'^2}{6}=\frac{s.\,b.\,d^2-s'b'd'^2}{6}$$

in which s' = the strain per sectional square inch at the distance $\frac{d'}{2}$ from the neutral axis, which is to s as d' is to d.

If the difference $d\,d'$ is very small compared to d, it may be near enough for practical purposes to consider the moment of resistance of one of the parts shaded in the figure as equal to its direct resistance multiplied by the distance of its centre of gravity from the neutral axis, and if the centre vertical member is of small proportionate area, it is neglected in determining the resisting moment of the section.

I will take a fair practical section of such a beam, to see how near the truth the approximation will be. The external portions are 15 inches wide by 1 inch thick, the angle pieces being 3 inches along the back of each limb by $\frac{1}{2}$ inch wide, and the vertical element $\frac{1}{4}$ inch wide.

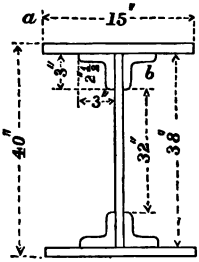

Fig. 13.

This latter by the first method is neglected in the calculation. Determining the centre of gravity of the section as before, we have, taking the upper boundary as the axis of moments—

$$\begin{aligned} 15 \times 1 \times \tfrac{1}{2} &= 7{\cdot}500 \\ 6 \times \tfrac{1}{2} \times 1\tfrac{1}{4} &= 3{\cdot}750 \\ 2\tfrac{1}{2} \times 1 \times 2\tfrac{3}{4} &= 5{\cdot}875 \\ \hline & 17{\cdot}125 = \text{moment of area } 20{\cdot}5. \end{aligned}$$

$$\frac{17{\cdot}125}{20{\cdot}5} = 0{\cdot}835 \text{ inches},$$

which, deducted from 20 inches, half the depth, leaves 19·165 inches for the distance of the centre of gravity of the area $a\,b$ from the neutral axis. This area is called the

area of the flange, the top and bottom elements of the beam or girder being known as the flanges.

Taking the working strain at 4 tons per square inch, the sum of the moments of resistance of both the flanges will be—

$$2 \times 19{\cdot}165 \times 4 \times 20{\cdot}5 - 3143{\cdot}06 \text{ inch tons.}$$

This has to be compared with the moment of resistance found by exact calculation from the formula—

$$M = \frac{s\ b\ d^2 - s'\ b'\ d'^2 - s''\ b''\ d''^2 - s'''\ b'''\ d'''^2}{6}.$$

The values of the second, third, and fourth strains must first be found; they will vary with the values of d—

$$s' = s.\frac{d'}{d},\ s'' = s.\frac{d''}{d},\ s''' = s\frac{d'''}{d}.$$

Replacing the s, s'', &c., by their values, we have

$$M = \frac{s}{b}\cdot\left(\frac{b\ d^3}{d} - \frac{d'}{d}\cdot b'\ d'^2 - \frac{d''}{d}\cdot b''\ d''^2 - \frac{d'''}{d}\cdot b'''\ d'''^2\right) =$$

$$\frac{s}{b.\ d}\left(b\ d^3 - b'\ d'^3 - b''\ d''^3 - b'''\ d'''^3\right)$$

whence—

$$M = \frac{4}{6 \times 40}\cdot\left(15 \times (40)^3 - 8{\cdot}75 \times (38)^3 - 5 \times (37)^3 - 1 \times (32)^3\right)$$

$= 3213{\cdot}95$ inch tons — being somewhat in excess of the approximate figures.

It should be noticed that if the working strain is taken at the centre of gravity of the flange, that of the fibres beyond that point is somewhat higher; not that this usually is of any practical importance, but still no matter should be overlooked in dealing with questions of principle.

If in the two flanges we are using different materials, so that there is not the same strain per sectional square inch on both, then this discrepancy must be made up by varying the areas of the flanges in order that the total resistance shall be the same.

If, however, homogeneous material is used throughout

the girder, and the flanges are not of equal area, they will adjust the internal strains so as to produce equal moments of strain about the neutral axis, which will be at some intermediate point depending on the ratios of the areæ. For example, let a girder have one flange 3 inches by 2 inches, and the other 12 inches by $1\frac{1}{2}$ inches, and let us assume the elastic resistances to tension and compression equal per sectional square inch; that is, that the modulus of elasticity is constant for both extension and shortening. The depth of the girder shall be taken as 20 inches, and the web omitted in the calculation. Let the distance from the centre of gravity of the top, which is the smaller flange, be x from the neutral axis, and that of the bottom or larger flange y.

If s = strain per sectional square inch on the top flange, that on the bottom flange will be equal to s multiplied by the area of the top and divided by that of the bottom flange.

The neutral axis is seen to be the fulcrum about which the moment of strain in one direction and the two moments of resistance in the other act, and it must so adjust itself as to suit the equilibrium of the three forces. This may be illustrated by considering the forces to act on the angles of a frame, as shown in Fig. 14. *a b e* represents the frame, *b e* being in the position of a section of the beam, having the neutral axis lying somewhere between *b* and *e*, and it must occupy such a position that under the three forces given the frame shall be at rest.

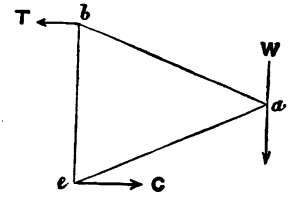

Fig. 14.

We know the *sum* of the moments of resistance, because that must be equal to the moment of strain, and the moment of strain is given, being W multiplied by the distance of its direction from the vertical line *b e*, and as its

direction is vertical, the position of the neutral axis higher or lower on *b e* will not alter the moment of force. When the force W begins to act, it pulls the point *b* and thrusts *e*, the amount of relative travel of these points depending upon the resistances offered by them; but when equilibrium is established, the gross stress T must equal the gross stress C; for if we regard the moment of W acting about C as a fulcrum to produce direct strain at T, or about T to produce direct strain at C, the result is the same as regards magnitude of the resulting direct strain, for in both cases the moment of strain is divided by *b e*, the depth of the girder.

In the case taken the areæ are as 3 to 1; hence the strains per sectional square inch will be in this ratio, and this will also be the ratio of the distances of the centres of gravity of the two flanges from the neutral axis; hence $x=15$ inches and $y=5$ inches, and this places the neutral axis in the centre of gravity of the entire section.

We can always then readily find the position of the neutral axis by determining the centre of gravity of any section with which it is necessary to deal.

The moments of resistance for the above section will be for the top and bottom flanges respectively, $a\ 3\times2\times s\times x$ and $12\times1{\cdot}5\times\frac{s}{3}\times y$, and the total moment of resistance is

$=6\times4\times15+18\times\frac{4}{3}\times5=360+120=480$ inch tons.

If this be accurate, it should be equal to the moment of resistance of the top flange about *e*, or of the bottom flange about *b*; the former is found to be $6\times4\times20=480$ inch tons; the latter $18\times\frac{4}{3}\times20=480$ inch tons.

I will now collect together the three formulæ for determining the moments of resistance as ascertained above.

For a form of section symmetrical in respect to the

neutral axis, such as that shown in Fig. 13, the general expression assumes the form—

$$M=\frac{s}{b\,d}\left(b\,d^3-b'\,d'^3-b''\,d''^3-\ldots\ldots\ldots-b^n\,d^{n3}\right)$$

where there are n spaces to deduct from the total moment of resistance of the circumscribing section.

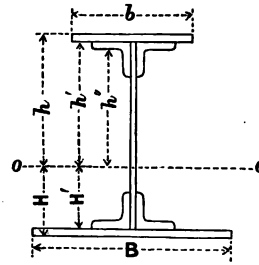

Fig. 15.

In an unsymmetrical section, like that of Fig. 15, $o\ o$, the neutral axis, is at the centre of gravity, and h is the greatest distance of any fibre from it; then—

$$M=\frac{s}{3\,h}\left(b\,h^3-b'\,h'^3-b''\,h''^3-\ldots\ldots\ldots-b^n\,h^{n3}\right)$$

If we are taking the flanges only of the girder, d being measured between the centres of gravity of those flanges, and A = area in square inches of smaller flange,

$$M=s\,.\,A\,.\,d\,.$$

Having investigated the nature and action of the internal resistances of the material, that of the external forces next presents itself, and this I shall treat in detail for the various forms in which it occurs, premising that in this chapter beams either of solid section throughout, or at least having a continuous web, will alone be dealt with.

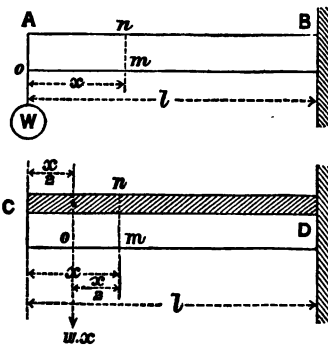

Fig. 16.

In Fig. 16, A B represents a beam or cantilever, one

end of which is fixed into a wall, the other end supporting weight, W. l is the length of the beam, measured from the wall to the point of attachment of the weight W. The moment of strain about any point will evidently be equal to the weight multiplied by its horizontal distance from that point; so if the point be distant x ft., the weight being in tons, the moment is—

$M=Wx$ foot tons; if $x=l$, $M=Wl$ foot tons,

which is the maximum strain that can come upon the beam, as l is the greatest value to which x can attain. At A, the point at which W is attached, the moment of strain is *nil;* but there it commences, and gradually increases to the point of support. As the strain thus varies, so the section of the beam may be varied to meet it. Equating this moment of strain with the moment of resistance of the flanged girder—

$$M=Wx=s \cdot A \cdot d \therefore A=\frac{W \cdot x}{s \cdot d},$$

whence the other quantities being given, the necessary sectional area of flanges is found. The beam may be adapted to the varying strain by varying either its area or its depth; but this question will be dealt with subsequently, after the theory of the external forces has been fully discussed.

Throughout the ensuing cases the moments of strain will be equated with the moment of resistance of the flanged girder, that being the description almost invariably used for beams of any magnitude.

At C D, Fig. 16, is shown a similar beam fixed at one end, and carrying a load uniformly distributed along its length (such a load might be its own weight, or a wall, &c.). Let the load be w per foot of length. The moment of strain is required at any point distant x from the free end, C, of the girder.

The load affecting this point will evidently be that part of the whole load which is between the given point and

the end of the beam; this length being x ft., the load upon it is $w \times x$, and this load being symmetrical in form, may be regarded as concentrated at the centre of its length, which is its centre of gravity, and is horizontally distant $\frac{x}{2}$ from the given point at which the moment of strain is sought. There is then a load equal to $w \,.\, x$ acting at a distance $\frac{x}{2}$; hence the moment of strain is—

$$M = w\, x \times \frac{x}{2} = \frac{w\, x^2}{2}, \text{ if } x = l,\ M = \frac{w\, l^2}{2},$$

and—

$$M = \frac{w\, x^2}{2} = s \,.\, A \,.\, d \ \therefore\ A = \frac{w\, x^2}{2 \,.\, s \,.\, d}.$$

The action of the load in producing strain upon the flange may be pictured to the imagination by supposing $o\ m\ n$ to be a bent lever acting about the fulcrum m, having at one end, o, the pull of the load, and at the other the elastic resistance of the material of the flange. In the first case the horizontal arm of the lever is x, and the vertical d; and in the second, the horizontal arm is $\frac{x}{2}$, and the vertical arm d.

As the strain increases as x^2, and l is the greatest value of x, the maximum moment is when $x = l$.

The rate of increase of strain differs in the two cases, being in the first as x, and in the second as x^2.

If the two loads occur at once, the resulting moments of strain must be added together to get the total moment: then—

$$M = Wx + \frac{w\, x^2}{2} = s \,.\, A \,.\, d \ \therefore\ A = \frac{1}{s \,.\, d}\left(Wx + \frac{w\, x^2}{2}\right)$$

and as a general rule, where a number of different loads come upon the same elements, their effects may be calcu-

lated separately for any given point, and the moments thus found added together for a resultant total.

In Fig. 17 are shown some beams variously loaded, each beam being freely supported — that is, the ends are not fixed down to the beds— at each end. In the case of A B the load W is at the centre of the span l. Now the load on a beam is carried by the supports, and from the elastic nature of materials, it imperatively follows that these points of support must be compressed by the superincumbent load, such compression continuing until the elastic resistance or reaction is equal to the load pressing downwards; and as the load does press vertically downwards, so the reaction of the supports must press vertically upwards against the ends of the girder.

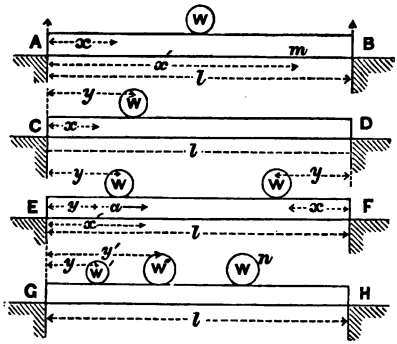

Fig. 17.

Where the load is in the centre of the span, it is evident that it will be carried only half on each support; hence the downward pressure on one support is $\frac{W}{2}$, and as the reaction must be the same, the upward pressure of the support is $\frac{W}{2}$.

The moment of strain, then, at any point distant x from the nearest support, is this reaction multiplied by the distance at which it acts, that is, by x, giving—

$$M = \frac{W}{2} \times x = \frac{W\,x}{2} = s \,.\, A \,.\, d \;\therefore\; A = \frac{W\,x}{2 \,.\, s \,.\, d}.$$

But if the calculations be continued, with x carried on beyond the load, so that it is not measured from the *nearest* support, then there will be two moments to determine, for beside the upward force of the reaction at the support, there is the downward force of the weight W to consider. Let the point m be in such a position, and distant x ft. from the support A. The distance of m from B will be $l-x$, and the distance of m from W will be half the length less this distance, or $\frac{l}{2}-\left(l-x\right)=-\frac{l}{2}+x$, or $x-\frac{l}{2}$, which is the distance at which the weight W acts about the point m.

Calling the forces acting downwards plus or positive forces, and those acting upwards minus or negative, the expression for the moment about m becomes—

$$M=W\times\left(x-\frac{l}{2}\right)-\frac{W}{2}\times x=Wx-\frac{Wl}{2}-\frac{Wx}{2}=\frac{Wx}{2}-\frac{Wl}{2}=$$
$$\frac{W}{2}\left(x-l\right)$$

but the difference between x and l is the distance from m to B, the nearest support; hence this formula corroborates the former, for measuring from B, we replace the $x-l$ by $-x$, and obtain—

$$M=-\frac{Wx}{2},$$

the minus sign coming in as the upward forces are called negative; but of course this does not affect the numerical value involved.

By adopting some such system as this and adhering to it, the nature of the strains on the flanges will be indicated by the sign in front of the formula; thus in this notation a minus sign indicates that the bottom flange is in tension and the top in compression, the beam being so bent downwards that the under side is convex, and the upper concave. This notation will be retained throughout the work.

C D shows another mode of loading, there being a concentrated weight at some point not at the centre of the span. The first step is to determine the reaction on the support from which x is to be measured. Let C be the support chosen, and the weight W be distant y ft. In order that equilibrium may be maintained, the moment of the weight W about the point D must be equal to the moment of the reaction on C about the same point, the former being plus and the latter minus in sign; the distance at which W acts about D is $l-y$, and the distance at which R, the reaction, acts about D is l; hence the moments are respectively—

$$-\mathrm{R}\times l \text{ and } \mathrm{W}\left(l-y\right) \text{ or } -\mathrm{R}\,l=\mathrm{W}\left(l-y\right) \quad -\mathrm{R}=\frac{\mathrm{W}(l-y)}{l};$$

and the moment of this reaction about any point distant x (x being less than y) from C $=-\mathrm{R}\,x=-\mathrm{M}=\frac{\mathrm{W}(l-y)\,x}{l}$.

$$\mathrm{M}=-\frac{\mathrm{W}(l-y)\,x}{l}=s\,.\,\mathrm{A}\,.\,d\;\therefore\;\mathrm{A}=-\frac{\mathrm{W}(l-y)\,x}{s\,.\,d\,.\,l}.$$

By taking a case where x is greater than y, the formula will be proved as was the last, but the process being precisely analogous, I shall not here occupy space with it.

At E F another special kind of loading is shown, and one which is of importance, as it occurs very commonly in practice. Here there are two equal weights symmetrically disposed in respect to the centre of the girder. This being the case, the loads on, and the upward reactions of the two supports will be equal, each being equal to one of the loads, W. Let the distances of the loads from the ends of the girder be y in each case; then, if $x=$ the distance from the nearest support to a given point between that support and the weight nearest it, the moment of strain will be

$$M = -W \cdot x = s \cdot A \cdot d \therefore A = -\frac{W \cdot x}{s \cdot d};$$

and this moment will go on increasing as x increases, until $x=y$, and if it be taken at a higher value still, then there will be two moments to deal with.

Let $x'=y+a$, then

$$M = W\,a - W\,x',$$

because a is the distance at which the weight W acts downwards about the given point, but

$$W\,a - W\,x' = W\,a - W\,(y+a) = W \cdot (a-y-a) = -W\,y.$$

Hence the moment of strain cannot exceed $W\,y$, and will remain at that value for all the length of girder between the two weights W.

The reason is found by inspection to be that whatever quantity above y is added to the distance at which the reaction acts about the given point, such addition will be the distance at which the weight itself acts in the opposite direction about the same point. Here then is a case where the strain on the girder is *not affected by the value of the span of the girder*—which will be found unique.

In the fourth case, G H shows a girder having a number of concentrated weights upon it, scattered irregularly along its length, and being of various values. Let the weights be represented by W′, W″, &c., W^n placed at distances y', y'', &c., to y^n along the girder. First the total reaction on one support must be determined, and then any given point being known, the weights between such point and the support chosen will have plus moments, the reaction of the support of course giving a minus moment.

The moment of the reaction of the support must equal the sum of the moments of all the weights about the other support, of course with the difference of signs, thus—

$$-R\,l = W'(l-y') + W''(l-y'') + \ldots + W(l^n - y^n)$$

$$-R = \frac{W'(l-y') + W''(l-y'') + \ldots + W''(l-y^n)}{l}$$

and the moment of strain at a point distant y^n from the chosen support will be

$$M = W'(y^n - y') + W''(y^n - y^2) + \ldots + W^{n-1}(y^n - y^{n-1}) - R y^n.$$

The intermediate moments may be found from this expression by replacing y^n by x, where x *exceeds* y^{n-1}, but does *not exceed* y^n.

As the W's and the y's do not follow any law in their changes, this formula must be worked out in detail from the quantities occurring in each particular case.

The maximum strain has not yet been determined for the case of A B, where the load is central; hence it will be necessary to return to it. It is observed from the formula $M = -\frac{W x}{2}$, that up to the centre of the span the load increases as x; it remains to be seen what will occur when $x =$ more than $\frac{l}{2}$, and when two moments must be dealt with.

Let $x' = \frac{l}{2} + a$, then a will be the distance at which W acts downwards about the given point, and the resultant moment of strain will be

$$M = W.a - \frac{W x'}{2} = W a - \frac{W}{2}\left(\frac{l}{2} + a\right) = W\left(a - \frac{l}{4} - \frac{a}{2}\right)$$

$$W\left(\frac{a}{2} - \frac{l}{4}\right) = -W\left(\frac{l}{4} - \frac{a}{2}\right)$$

whence it is evident that the arithmetical value decreases as a increases, or, as soon as the value $x = \frac{l}{2}$ is passed, the moment of strain diminishes; hence the maximum strain is when $x = \frac{l}{2}$, and

$$M = -\frac{W x}{2} = -\frac{W l}{4} = s.\,A.\,d. \therefore A = -\frac{W l}{4.\,s.\,d.}$$

It will be observed that in the expression $\frac{l}{4}-\frac{a}{2}$, a can never exceed $\frac{l}{2}$, and consequently the highest value of a gives

$$\frac{l}{4}-\frac{a}{2}=\frac{l}{4}-\frac{l}{4}=o.$$

The strain is $=o$ at the support B.

The second case, C D, now requires examining as to the point of maximum strain. It is evident that the moment of strain continues to increase as x increases up to the value y. Now let $x'=y+a$, then the moment of strain

$$M=W\,a-\frac{W(l-y)x'}{l}=W\,a-\frac{W(l-y)}{l}(y+a)=W\,a$$
$$-\frac{W(l-y)y}{l}-\frac{W(l-y)a}{l}.$$

From this we may observe what will be the effect of increasing the value a. In the first term, $W\,a$, the term increases as a increases, and in like manner the third term increases as a increases, but $\frac{l-y}{l}$ is less than unity, therefore $\frac{W(l-y)a}{l}$ must always be less than $W\,a$. The ratios of the two quantities being constant, the absolute difference between them will increase, so that after passing the loaded point the minus strain will continually diminish towards the support D; this diminution will continue until D is reached, when $a=l-y$, replacing a by this value.

$$M=-W(l-y)-\frac{W(l-y)y}{l}-\frac{W(l-y)(l-y)}{l}$$
$$=W\,l-W\,y-W\,y+\frac{W\,y^2}{l}-W\,l+2W\,y-\frac{W\,y^2}{l}=o,$$

there being no strain over the points of support.

From this it is evident that when a beam is loaded with

one concentrated weight, the maximum strain will be at a point directly under that weight.

Let A B, Fig. 18, represent a girder supported freely at A and B, and loaded with a weight of w tons per lineal

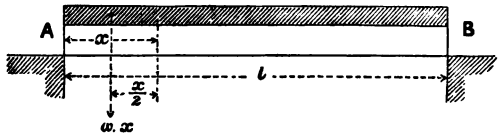

Fig. 18.

foot, uniformly spread over its length. The strain is required at a point distant x from the support A. There is the portion of the load between the given point and A acting downwards, or positively, and the reaction of the support A acting upwards, or negatively. As the load is uniformly distributed along the length of the girder, it will be equally carried by the two points of support. The reaction of either support will therefore be

$$\mathrm{R}=-\frac{w.\,l}{2},$$

and this reaction will act at a distance x about the given point. The downward force will be the portion of the load on the length x of the girder, or $w\,x$, and this being considered as concentrated at its centre of gravity, will act at a distance $\frac{x}{2}$ about the given point. Taking, then, the difference of the moments, the resultant is

$$\mathrm{M}=w\,x\times\frac{x}{2}-\frac{w\,l}{2}\times x=\frac{w\,x^2}{2}-\frac{w\,l\,x}{2}=s.\,\mathrm{A}.\,d$$

$$\therefore \mathrm{A}=-\frac{w}{2.\,s.\,d}(x^2-l\,x),$$

considering the variable part of the expression $x^2-l\,x$ will lead to the determination of the point of maximum strain.

When $x=o$ the strain $=o$ at A, and when $x=l$ the equation becomes $l^2-l^2=o$ and the strain $=o$ at B; hence there is an intermediate point, at which the strain is a maximum.

The loading being uniform (and symmetrical), the increase of the strain will be in the same ratio from the end B as from the end A, and the maximum strain must occur, therefore, at a point equidistant from both supports, which is at the centre, when $x=\frac{l}{2}$ and

$$M=\frac{w\,x^2}{2}-\frac{w\,l\,x}{2}=\frac{w\,l^2}{8}-\frac{w\,l^2}{4}=-\frac{w\,l^2}{8}=s.\,A.\,d.\therefore A=-\frac{w.\,l^2}{8.\,s.\,d}.$$

The total load is $W=w\,l$, so that $M=-\frac{W\,l}{8}$ as against $-\frac{W\,l}{4}$ when the weight was all at the centre, whence it appears that the maximum moment of strain due to a

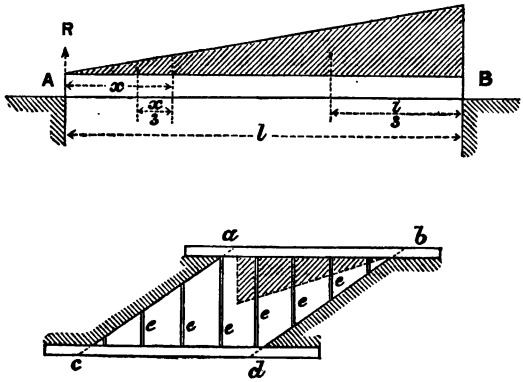

Fig. 19.

uniformly distributed load is one-half of that due to the same load concentrated at the centre of the span.

A B, Fig. 19, represents a girder carrying a load distributed in the form of a triangle: this case is a very important one, as in practice it constantly occurs when the girders of a bridge do not lie square to the abutments, as shown in the plan *a b*, *c d*. The road under the bridge runs obliquely to that over it. The bridge is carried by cross girders, *e*, resting on the main girders *a b*, *c d*, and it may be observed that the main girder carries a triangular loaded area, which is shaded in the figure.

The load per lineal foot will not be the same throughout the length of the girder, but will vary with x, and in direct ratio to it.

Let w' = the load on the first lineal foot, then at the distance x the load per lineal foot will be $w' x$; hence the load on a part of the girder extending from the support A to the distance x will be $w' x^2$, and this load being triangular in disposition, will have its centre of gravity $\frac{2x}{3}$ from A, so that the distance at which it acts about a given point at x from the support A is $\frac{x}{3}$, thus making the positive moment of strain $w' x^2 \times \frac{x}{3}$.

The total load on the girder is $w' l^2$, for as the load up to any point distant x from A $= w' x^2$, the total load is found by making $x = l$; then it is $w' x^2 = w' l^2$; and as the centre of gravity of this load is $\frac{l}{3}$ from the support B, the moment of the total load about B is $w' l^2 \times \frac{l}{3}$. The moment of the reaction of A about B is $-\mathrm{R} \times l$; hence the reaction is—

$$\mathrm{R} = w' l^2 \times \frac{l}{3} \times \frac{1}{l} = \frac{w' l^2}{3},$$

and its moment about the given point is this quantity

multiplied by x; the resultant moment of strain on the beam is—

$$M = w' x^2 \times \frac{x}{3} - R.\, x = \frac{w' x^3}{3} - \frac{w'\, l^2\, x}{3} = \frac{w'}{3}\,(x^3 - l^2\, x) = s.\ A.\ d.$$

$$\therefore A = \frac{w'}{3\ s.\ d.}\,(x^3 - l^2\, x).$$

In this case the areæ for different values of x must be calculated out for the whole length of the girder, which will not be symmetrical in the distribution of strains from each point of support. In the girders symmetrically strained it is sufficient to calculate the strains for one half, as the two halves have similar strains upon them. It will be interesting to ascertain the point of maximum strain.

The strain continues to increase up to the point of maximum strain, and then to decrease according to the formula $\frac{w'}{3}(x^3 - l^2\, x)$; that is, in the same ratio as $x^3 - l^2\, x$. Now, as there is a point at which the strain ceases to *increase* and commences *diminishing*, it may be imagined to remain stationary for indefinitely slight variation of x at that point.

Let x' = distance from A corresponding to point of maximum strain; let a be an indefinitely small increase added to x; now, if the strain remains the same, the numerical value of the increase of the first term x^3 must equal that of the second $l^2\, x$, in order that their difference may not be altered. As x becomes $x' + a$, the first term will be $(x' + a)^3 = x'^3 + 3x'^2\, a + 3x'\, a^2 + a^3$; but as a is taken very small, $3x'\, a^2$ will be very insignificant compared with $3x'^2\, a$, and a^3 less still; hence these last terms may be neglected, leaving $3x'^2\, a$ as the amount to be added to the first term in $x^3 - l^2\, x$. The second term then becomes $-\, l^2\, (x' + a) = -(l^2\, x' + l^2\, a)$; here the addition is $l^2\, a$, and this must be equal to the addition $3x'^2\, a$ if the strain is unaltered, thus—

$$3x'^2 a = l^2 a . \therefore 3x^2 = l^2, \; x^2 = \frac{l^2}{3}, \; x = \frac{l}{\sqrt{3}} = 0{\cdot}577\, l,$$

and the maximum strain is—

$$M = \frac{w'}{3}(x^3 - l^2 x) = \frac{w'}{3}\left((0{\cdot}577\, l)^3 - 0{\cdot}577\, l^3\right) = 0{\cdot}1283\, w'\, l^2$$

$$= s.A.d . \therefore A = \frac{0{\cdot}1283\, w'\, l^2}{s.d}.$$

It is unnecessary to consider the cases of partial loads, as in practice the total loads are required, for the structure must be proportioned to sustain the maximum strain, and this covers all beneath it.

These beams have all been treated as freely supported at the ends, in which circumstance the intensities of the strains are quite independent of the material or form of the girder, but if the ends be not free a striking difference is observed.

Let the values of M be calculated for a series of values of x, and laid down as shown in Fig. 20. A B is a line

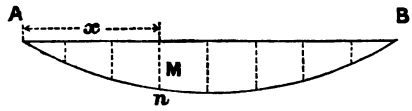

Fig. 20.

representing the length of the beam, and on this line the various values of x are marked off from A, and the values of M, corresponding to those values of x, are at each point marked off at right angles to A B, giving points of which one is indicated at n; by joining these points a curve is formed, which is called the curve of strain, and the lines showing the values of M are called ordinates to that curve.

A geometrical investigation would show that for a uniformly distributed load the curve of strain is a parabola;

for a concentrated load the lines of strain bound a triangle, and so forth, the lines varying according to the description of load.

If the whole area of the curve be taken and divided by its length, it is evident the average or mean moment of strain will be the result.

As the area of a parabola is its base multiplied by two-thirds of its height, the mean strain on the uniformly loaded girder will be two-thirds of the maximum strain. The use of these curves when plotted will presently become evident. In Fig. 21 let A B represent a beam securely

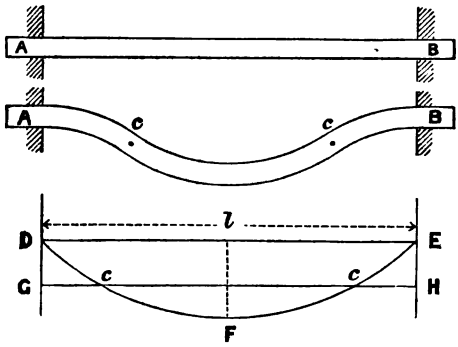

Fig. 21.

fixed at both ends, so that the section at A is retained in a vertical plane, and the same at B. If, now, the beam be deflected, it will take a form A *c c* B, there being two changes in the direction of curvature, for while the centre part is concave on the upper surface the end parts will be concave on the under surface, so that at the points of contrary flexure the nature of the strains on the flanges will change.

Let D E represent the line of a beam free at the ends, and D F E the curve of strain upon it. If, now, the curve

is considered as applied to a beam with fixed ends, it is evident something must be deducted from the ordinates for the reversed strains commencing at the points of contra-flexure. Let D G and E H represent the amount of the moments of strain at D and E, join G H, then the strains on the central part will be the ordinates to the curve c F c measured downwards from the line G H, and the strains on the ends will be the ordinates measured upwards from G c and H c to the curves D c and E c. The whole system will in effect consist of a central beam supported freely at its ends c c, and two cantilevers or brackets, G c, H c, sustaining it and the load upon themselves, if any. The question here to be determined is the position of the points c c.

Assuming the beam to be of uniform section, and the tensile and compressive strains to divide themselves equally on the flanges—that is to say, that the totals shall be equal—then the area c F c will be equal to the sum of the areæ D G c, E H c. The area of a parabola is its base multiplied by two-thirds of its height; hence, taking D E as uniformly loaded, we can find the required positions for c c.

By the formulæ preceding for maximum moments of strain, it is shown that the centre or maximum ordinate of D F E is $\frac{w\,l^2}{8}$; call $w = 1$, then this becomes $\frac{l^2}{8}$.

Let z = G c = c H . a = area c F c; b and b = areæ D G c and E H c; then as these areæ represent the sums of the strains on each part—

$$a = b + b.$$

The greatest ordinate of the curve c F c is $\frac{(l - 2z)^2}{8}$, where $(l - 2z) = c\,c$ is the effective span of the central beam. The area $a = \frac{(l - 2z)^2}{8} \times \frac{2}{3} \times (l - 2z)$; $(l - 2z)$ being the base of the parabola c F c. The sum of the areæ

$b + b$ will be D G H E less D c c E, and D c c E will be the area D F E less the area c F c; therefore

$$b + b = l \times \left(\frac{l^2}{8} - \frac{(l-2z)^2}{8}\right) - \frac{l^3}{12} + \frac{1}{12}(l-2z)^3,$$

where l = side G H of rectangle D G H E; $\left(\frac{l^2}{8} - \frac{(l-2z)^2}{8}\right)$ difference between maximum ordinates to D F E and c F c, which equals the height D G of the rectangle, $\frac{l^3}{12} = \frac{l^2}{8} \times \frac{2l}{3}$ = area D F E, and the last term equals the area c F c

$$= \frac{(l-2z)^2}{8} \times \frac{2}{3}(l-2z),$$

but $a = 2b$,

$$\frac{(l-2z)^3}{12} = l \times \left(\frac{l^2}{8} - \frac{(l-2z)^2}{8}\right) - \frac{l^3}{12} + \frac{(l-2z)^3}{12}, \therefore$$

$$\frac{4l^2z - 4lz^2}{8} = \frac{l^3}{12},$$

$$\text{whence } z^2 - lz = -\frac{l^2}{6}.$$

In this expression z^2 l z is termed an imperfect square, and in order to solve the equation it must be completed, and whatever is necessary to complete it must also be added to the other side of the equation, that the two sides may remain of equal value. We must see what is required to complete the square. Let a b c d, Fig. 22, be a square, supposed to represent the square we seek. Let $a\,b = a\,c = z$. Let the square root of a b c d be $z - y$; if this be now squared it becomes $z^2 - 2zy + y^2$, and in the figure these areæ are thus shown: $z^2 = a\,b\,c\,d$, $y^2 = h\,e\,b\,f$, $(z-y)^2 = g\,c\,i\,e$; $g\,c\,i\,e =$

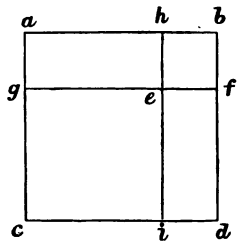

Fig. 22.

$abcd - agfb - eidf$, or instead of the last term $-(hidb - hefb)$, $gcie = abcd - 2agfb + hefb$. In our equation— lx corresponds to $2zy$, and $2y$ corresponds to l; hence y^2 is the part to be added to complete the square, but $2y = l$, $y = \frac{l}{2}$; hence $\left(\frac{l}{2}\right)^2$ must be added to each side of the equation, and we get—

$z^2 - lz + \frac{l^2}{4} = \frac{l^2}{4} - \frac{l^2}{6} = \frac{l^2}{12}$, $\sqrt{z^2 - lz + \frac{l^2}{4}} = \sqrt{\frac{l^2}{12}}$, $z - y$ or $z - \frac{l}{2}$ has been seen to be the root of the square we have completed; hence $z - \frac{l}{2} = \pm \frac{l}{\sqrt{12}}$. We put plus or minus as either sign multiplied by its like gives a positive quantity, and therefore the root of a^2 may be either $+a$ or $-a$. $z = \frac{l}{2} \pm \frac{l}{3{\cdot}464} = 0{\cdot}79l$, or $0{\cdot}21l$ (nearly). These two lengths will therefore give the position of the points of contraflexure, and the strain at the centre of the span will be

$$M = -\frac{w(l-2z)^2}{8} = -\frac{wl^2}{24}.$$

The strains over the points A and B are each

$$M = \frac{wz^2}{2} + \frac{w(l-2z)}{2} \times z = \frac{wl^2}{12}.$$

Suppose a case occurs in which the beam is *fixed* at one end only, being freely supported at the other, then there will be one point of contrary flexure, and it will be the same as if we take the part G c c of the beam G H. What then will be the relation of G c to the span G c c? G c c $= l - z = l - 0{\cdot}2113\,l = 0{\cdot}7887\,l$, and $\frac{0{\cdot}2113l}{0{\cdot}7887l} = 0{\cdot}26$. Or in this case $z = 0{\cdot}26\ l$, where l is the length of the beam fixed at one end and freely supported at the other.

Let us look at the first case from another point of view to render the result susceptible to proof. Suppose the beam first to be freely supported at A and B. Then the

top flange will be shortened and the bottom one lengthened when the load is upon it, and will appear as shown in the sketch. To bring the ends back to a vertical plane, the

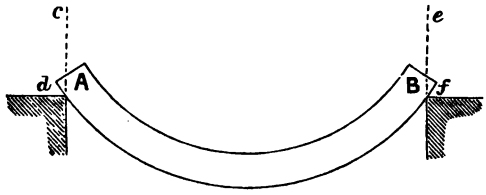

Fig. 23.

top flange must be as much extended as it has been shortened, and the bottom flange similarly compressed; but the strain to do this must evidently be equal to that causing the existing compression and extension, which is two-thirds of the maximum strain; hence the moment to be applied over each support (they will act in opposite directions as action and reaction) will be—

$$\frac{2}{3} \times \frac{w\,l^2}{8} = \frac{w\,l^2}{12}.$$

Applying this to the general formulæ for strain at any point on a uniformly loaded girder, this new moment being positive,

$$\mathrm{M} = \frac{w\,l^2}{12} + \frac{w\,x^2}{2} - \frac{w\,l\,x}{2}$$

which at the points of contra-flexure will be—

$$\mathrm{M} = o = \frac{w\,l^2}{12} + \frac{w\,x^2}{2} - \frac{w\,l\,x}{2} \quad \therefore \frac{w\,x^2}{2} - \frac{w\,l\,x}{2} = -\frac{w\,l^2}{12},$$

whence $x^2 - l\,x = -\frac{l^2}{6}$, being the same equation as resulted from the former entirely different mode of reasoning.

It is observable that accuracy here depends upon whether the material is homogeneous throughout, for if its

modulus of elasticity be not constant these results will not be strictly attained.

So much for the uniform section, but the commoner case is where the section of the girder is not uniform throughout, but on the contrary is varied in proportion to the strain, so that the strain per sectional square inch is constant throughout the length of the beam.

When the strain per sectional square inch is the same throughout, the beam will deflect in circular arcs of the same radius, and the points of contra-flexure will therefore occupy positions different from those assigned to them in the last case.

In Fig. 24, let *a b c d* be the line of beam under uniform

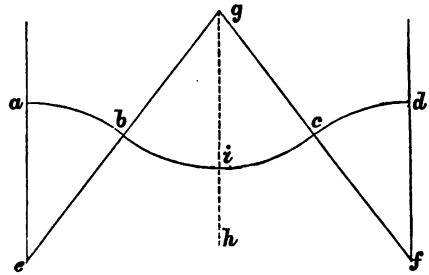

Fig. 24.

strain. The radii *e a*, *b g*, *c f* being all equal, it follows that the arc *a b* = arc *b i* = arc *i c* = arc *c d*, *g h* being drawn vertical and parallel to *a e* and *d f*. Here, then, the distance from one end of the beam to the nearest point of contra-flexure will be $\frac{l}{4}$.

In this case the same test cannot be applied as in the last; for the area varying to suit the strain, we cannot regard the beam in the first instance as merely supported, and then brought into the position of the fixed beam, as

the theoretical section being *nil* at *b* and *c*, the beam would not stand under the first conditions.

As the effective span of the central part is half the total span, or $\frac{l}{2}$, the strain at the centre will be

$$M = \frac{w}{8} \times \left(\frac{l}{2}\right)^2 = \frac{w\,l^2}{32} = s.\,A.\,d \therefore A = \frac{w\,l^2}{32.\,s.\,d.},$$

and that at the point of fixture—

$$M = w \times \frac{l}{4} \times \frac{l}{8} + w \times \frac{l}{4} \times \frac{l}{4} = \frac{3\,w\,l^2}{32} = s.\,A.\,d \therefore A = \frac{3.\,w\,l^2}{32.\,s.\,d.}.$$

The duty of the web or central part of flanged girders will now require examination, for it does not act merely as so much of the beam under longitudinal strain only, neither can it be considered as bearing shearing stress alone, but there is a vertical crushing, or rather crippling, strain brought upon it by the tendency of the flanges to approach each other when the beam is deflected.

Before the intensity of the strain thus called into action can be determined, a formula must be found for reducing a tangential force to its radial component.

In Fig. 25, let A represent a ring one foot deep, its diameter being equal to D. Assume this ring to be subject to internal pressure (as from steam, for instance), acting equally in all directions, and therefore pressing radially upon the interior of the ring. Considering the tendency to break the ring at the points *b c*, we find the forces acting will be those contained on the plane *b c*, which, acting from opposite sides of such imaginary plane against each other, tend to force asunder the hemi-rings *b d c*, *b e c*: *b c* = the diameter; hence, if P = the pressure per square foot, the stress on the

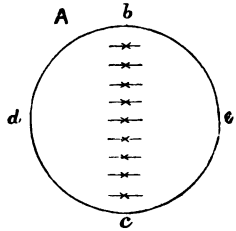

Fig. 25.

two parts b and c is $P \times D$, and the proportion borne by one section b or c of the ring is $P \times \frac{D}{2}$, or the force per square foot multiplied by the radius in feet of the ring.

From this it is evident that the tangential strain is equal to the radial strain multiplied by the radius, and this will apply to any part of the ring, for the points of strain might be taken at any other points than b and c, and moreover the strain on one part of the ring would not be affected were the rest cut away, provided the extremities of the part left are properly secured; hence the general formula —if on a curved element the force acts radially to such curve, the strain along the element will be the force per lineal foot multiplied by the radius in feet of the element at the place where such force is acting.

The web of the girder, acting to maintain the normal distance between the top and bottom flanges, will be subject to a radial pressure corresponding to the stress on and curvature of those flanges.

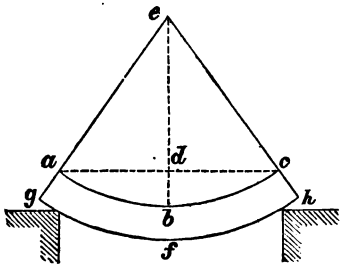

Fig. 26.

The total strain is the same for either flange, as has been shown previously (page 28), and this strain, divided by the radius of curvature of the deflected beam, will give the force upon the web tending to cripple it vertically. In order to determine the relation of this force to the longitudinal stress on the flanges, the radius of the deflected beam must be ascertained.

Assuming the beam to be so proportioned that its flanges

are equally strained throughout, let (in Fig. 26) $l = a\,b\,c =$ length of girder, $d = b\,f =$ depth of girder, $\mathrm{R} = e\,a =$ radius of curvature, $\mathrm{D} =$ central deflection $= d\,b$, $\mathrm{L} =$ difference in length of flanges after deflection = sum of the extension of the bottom flange and the compression of the top flange.

The deflection of a girder being so small in comparison with the radius of curvature, we may assume $d\,e = a\,e = \mathrm{R}$, and $a\,c = a\,b\,c = l$. Then (Euclid, prop. 35, Bk. III.) because, if any two right lines contained in a circle intersect one another, the rectangles formed by the segments of such right lines are equal, and the diameter 2 R is intersected at d by the chord $a\,c$, therefore $2\,\mathrm{R} \times \mathrm{D} = a\,d \times d\,c = \frac{l^2}{4}$.

Because $a\,d = d\,c = \frac{l}{2}$ and $\left(\frac{l}{2}\right)^2 = \frac{l^2}{4}$, whence $\mathrm{D} = \frac{l^2}{2\,\mathrm{R} \times 4} = \frac{l^2}{8\,\mathrm{R}}$. Then by similar triangles $\mathrm{R} : d :: l : \mathrm{L}$, which we also know to be true because the circumferences of circles are in direct ratio to their radii, whence $\mathrm{R} + d : g\,f\,h :: \mathrm{R} : a\,b\,c$, but $g\,h\,f = a\,b\,c + \mathrm{L}$, and $\mathrm{R} + d : a\,b\,c + \mathrm{L} :: \mathrm{R} : a\,b\,c$; whence, as $a\,b\,c = l$, we find in either case $\mathrm{R\,L} = d.l$ $\therefore \mathrm{R} = \frac{d\,l}{\mathrm{L}}$. But $\mathrm{D} = \frac{l^2}{8\,\mathrm{R}}$ $\therefore \mathrm{R} = \frac{l^2}{8\,\mathrm{D}}$, $\mathrm{R} = \frac{l^2}{8\,\mathrm{D}} = \frac{d\,l}{\mathrm{L}}$ $\therefore \mathrm{D} = \frac{\mathrm{L}\,l}{8\,d}$ which is the formula for the central deflection. If E = the modulus of elasticity of the material, and S and F = respectively the strains on the top and bottom flanges, per sectional square inch, and s = mean of the two strains,

$$\mathrm{R} = \frac{d\,l}{\mathrm{L}} = \frac{d.\,l}{l \times \frac{2\,\mathrm{S}}{\mathrm{E}}} = \frac{d\,\mathrm{E}}{2\,s},$$

and also generally

$$\mathrm{R} = \frac{d.\,\mathrm{E}}{\mathrm{S} + \mathrm{F}}.$$

If R be taken in feet, and also the other dimensions, and

E, S, and F in tons, and C be called the crippling force on the web per foot of length, P the total strain on one flange,

$$P = R \times C \therefore C = \frac{P}{R} = \frac{P}{\frac{d\,E}{S+F}} = \frac{P \times (S+F)}{d.\,E} = \frac{P.\,s}{d.\,E}.$$

For example, take a girder 100 feet span, 8 feet deep, having flanges equally strained, there being 4 tons per sectional square inch upon each flange, and let the load on the girder be 2 tons per lineal foot, then

$$P = \frac{w.\,l^2}{8.\,d} = \frac{2 \times (100)^2}{8 \times 8} = 312{\cdot}5 \text{ tons}; \; C = \frac{312{\cdot}5 \times s}{d.\,E}$$

If the material be wrought iron, and the modulus of elasticity taken at 6,400 tons—

$$C = \frac{312{\cdot}5 \times 4}{8 \times 6400} = 0{\cdot}0244 \text{ tons per lineal foot.}$$

This shows a result so small that in the generality of cases it will be more than amply covered by the margin necessary for the purposes of manufacture. The load carried by the girder must entirely or in part pass through the web on its way to the flanges. If the load be carried on the bottom flange, tensile strains will in the first place prevail in the web, and *vice versâ*. But the vertical strains arising thus can never *exceed* the girder load. In the above case the load being 2 tons, only half a square inch per foot is called for to sustain it.

The shearing strain at any point on a cantilever is evidently the weight between that point and the end of the beam, or $w \times x$. In beams supported at both ends, the shearing strain at any point will be, for a concentrated load, equal to the reaction of the nearest support; for a load uniformly distributed over the whole length, equal to the load per foot multiplied by the distance from the centre of the span, or if y = that distance, $w \times y$, so that the maximum shearing strain will be at the support where y is greatest. In the above case the maximum shearing strain

will be $w \times y = \frac{w\,l}{2} = \frac{2 \times 100}{2} = 100$ tons, requiring 25 square inches to carry it.

It will be found that in practice the webs are invariably made much thicker than is indicated by theory, but in all parts of built structures, such as wrought-iron girders, it must be remembered that it is at the weakest parts, *the joints*, that the areæ have to be fitted to the strains.

In connection with the subject of the deflection of girders, it is noticeable that the practical deflection is less than might be expected from the modulus of elasticity as determined from experiments on tensile resistance. The modulus of elasticity for wrought iron is given as 24,900,000 lbs. From some very carefully conducted experiments made at King's College, London, it was found that the modulus as determined from the deflection of wrought-iron bars averaged 27,500,000—a considerable increase. It has also been known ever since particular attention has been paid to iron as a material for construction, that the resistance of solid cast-iron beams shows an immensely greater amount of tensile strength under transverse than under direct strain.

Twenty-five years since, this matter was carefully investigated by Mr. W. H. Barlow, and then was set at rest the question of the actual position of the neutral axis, which was found to be at the centre of gravity of the section, and by subsequent experiments the same was found to hold good for wrought-iron beams. In the former investigation very wide differences were found to occur, the excess of tensile resistance in the beam experiments being more than the tensile strength of the metal under longitudinal strain, the results as determined from Mr. Hodgkinson's experiments on cast-iron beams being as follows, where T is the direct tensile resistance, and D the extra tensile resistance shown under transverse strain in lbs. :—

T. 18537, 18323, 18312, 18085, 19501, 17734, 20242.

D. 23171, 22904, 22890, 20606, 24626, 22167, 25302.

The ratios of D to T were found to average as 1 to ·78, and those of Mr. Barlow as 1 to ·81. That a similar thing occurs in the case of wrought iron was also observed, but that material is difficult to measure the compressive resistance of, because it does not crush like cast iron, but bends and becomes distorted before its ultimate strength is arrived at; but it is observed that, although there is a great difference between the *ultimate* tensile and compressive resistances of wrought iron, yet the force necessary to overcome the elastic resistance does not so vary.

Mr. Barlow referred this accession of strength to the molecular disturbances occasioned by the flexure of the beam, and termed the increment of strength the resistance of flexure.

The difference of action of the force is this: in direct longitudinal strain the layers of molecules during extension move together; one is not more extended than the next, but when a beam is bent, the outer layers are lengthened and shortened more than the inner, and therefore one layer moves upon another, and so gives rise to the resistance of flexure.

Applying our formula for transverse resistance to a cantilever 1 inch square and 1 inch long, we have, if we take the resistance from the experiments of direct strain, breaking weight at end $= \frac{s.\ b.\ d^2}{6} = \frac{18000 \times 1 \times 1^2}{6} = 3{,}000$ lbs., but find that 7,000 to 8,000 lbs. practically represents the ultimate strength of the cantilever.

The average of a number of experiments on cast-iron bars 1 inch square, supported with a bearing of 36 inches, gave as the mean breaking weight at the centre 844 lbs.

$$\frac{W\,l}{4} = \frac{s\,b\,d^2}{6} \therefore s = \frac{3\,W\,l}{2.\ b.\ d^2} = \frac{3 \times 844 \times 36}{2 \times 1 \times 1^2}$$

= 45,476 lbs. as the longitudinal resistance tensile and to flexure: the latter may be regarded as the elastic resistance to shearing stress.

The last figures give, for the resistance of a sectional square inch of the material to transverse breaking, 7,596 inch lbs.

The resistance of cast iron to shearing force has been found to be double that of its tensile resistance, and it would appear that the elastic resistance of flexure agrees with this, for the total movement in the bent beam of one layer upon another will average one-half of the total extension, and the resistance to flexure is shown to be roughly equal to the tensile resistance proper, which would correspond to a shearing resistance double that to tension.

With wrought iron, on the other hand, the shearing resistance is somewhat less than the tensile strength.

In flanged girders, where the flanges are thin compared with the depth, this resistance to flexure will diminish as the extension and compression of the fibres become sensibly uniform.

CHAPTER IV.

FRAMED STRUCTURES.

I AM now passing on to consider a class of structures wherein the strains act longitudinally upon the different component parts, the whole work consisting of bars framed together in such manner that they form, as it were, channels for the strains to pass along.

In calculating the strains recourse will be had to the principle of the parallelogram of forces already explained in Chapter II.

There are two characteristic dispositions of bars shown in Fig. 27, by the use or combination of which all braced works are built up. In the first, two bars, A B and A C, joined at A and resting on abutments at B and C, support between them a load, W. Each bar carries part of the *load as such* on to its abutment, the proportion carried in each direction depending upon the position of W in regard to B and C. If B d be a horizontal line between the abutments, extending from centre to centre of the feet of the bars B and C, and this be intersected at e by the vertical line joining A and the load W, then the part of the load on B will be $W \times \frac{e\,d}{B\,d}$, and that on C will be $W \times \frac{B\,e}{B\,d}$.

H, I, J represents another arrangement in which the whole load is carried on the abutment H by the bar I H; I J being *horizontal*, and therefore carrying no load, its duty

is to keep the end I of the bar I H from falling towards J.

Returning to the first arrangement, on A e mark off the distance A o (to any convenient scale) to equal the weight W; complete the parallelogram A p o q, making o p parallel to A B and o q parallel to A C. Then the thrusts on A B and A C will be represented by A q and A p respectively, and will pass away into the abutments in the directions f and g; the vertical component on each abutment being the part of the load already assigned to it. From q and p draw horizontal lines meeting A e in the points r and t, then A r and A q will be the vertical forces corresponding to the strains upon the bars A B and A C, for if B f be made equal to A q, and this oblique thrust on the abutment resolved into its vertical and horizontal components, by completing the parallelogram B u f v, u f or B v will be the vertical component, and because A r and B v are both vertical they are therefore parallel, and meeting the straight line A f at the points A and B, make the angle r A B equal to the angle v B f. Again, q r is drawn horizontal, and also f v; hence they are parallel, and meeting the line A f at q and f, make the angle r q A equal to the angle v f B, and the remaining angles f v B, q r A are equal; hence the triangles

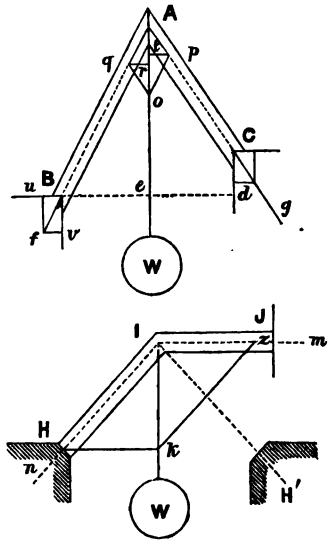

Fig. 27.

are similar, but the side Bf has been made equal to the side Aq; therefore the triangles Aqr, Bfv are equal, and their sides Ar, Bv are equal—that is, Ar is equal to the vertical load carried on the abutment B. And in like manner it can be shown that At is equal to the vertical load on the abutment C.

But, again, by similar triangles, Aq is to Ar in the same ratio as A B is to Ae, because Arq, AeB are both right angles, and the angles Aqr, A Be are equal; hence the strain on A B is to the weight it carries as the length A B is to the height Ae.

Now the length A B is the length of the bar carrying a certain proportion of load $\left(\frac{W \times ed}{B\,d}\right)$, and A$e$ is the difference in level between the top and bottom of the bar, or, in other words, Ae is the vertical height of the bar; hence we have a simple rule for the strain on an inclined bar when the vertical load carried by it is known, *i.e.* the strain on the bar is equal to its vertical load multiplied by the length of the bar and divided by its vertical height. All these dimensions must be measured on lines running along the centres of the elemental bars, as shown dotted in the figure.

In the present case, for example, let the load be 5 tons, the length of A B 7 feet, its vertical height 5·75 feet, the distance Bd 6·3 feet, and the distance ed 2·5 feet, then the part of the load on A B will be $W \times \frac{ed}{B\,d} = 5 \times \frac{2{\cdot}5}{6{\cdot}3} =$ 1·984 tons, and the strain upon the bar A B will be $1{\cdot}984 \times \frac{7}{5{\cdot}75} = 2{\cdot}415$ tons.

The same rule can, of course, be shown to apply to A C or to any other inclined bar, the vertical load upon which is known, or can be ascertained.

All that remains is to determine the *nature* of the strain

on the bar, and this is done by inspection. It is evident in this instance that it is in compression, running as it does from the top to the bottom of the bar; and thus the strain is determined as to its nature: if the load comes upon the highest part, and the bar is supported at its lowest end, the strain is compressive; but if the load is attached to the lower end of the bar, which is supported by its upper end, the strain is then tensile.

Next, we will examine the strains upon the second arrangement shown, where the whole load is carried by the bar A B.

The strain on I H is determined in the same way as that on A B, and is equal to $W \times \frac{H I}{I k}$, I k being a vertical line drawn from the top of the bar I H, and meeting the horizontal line drawn from the bottom in k. Let I k represent the weight W (to scale), and complete the parallelogram I H k s, then I s will be the horizontal strain upon the bar I J. But because H k, I s are opposite sides of a parallelogram, they are equal, and the horizontal strain on I J is equal to H k. We have then the strain on the horizontal bar in ratio to the load, the same as the ratio of H k to I k, but I k is the vertical height of the bar braced or sustained by I J, and H k is its horizontal projection from its foot; hence, if a bar carrying a load be retained in position by a horizontal member, the strain on that horizontal member will be equal to the load multiplied by the horizontal projection, and divided by the vertical height of the inclined bar.

In the present case let the load be 8 tons, H k 4 feet, and I k 6 feet, then the horizontal strain will be $8 \times \frac{4}{6} =$ 5·33 tons. Here we see the strain on I H is compressive, as is also that on I J, which is pushed against the wall at s, but if the inclined bar were in the position shown by

the dotted line I H′, the horizontal member would be in tension; hence, in such an arrangement, if the inclined bar is in compression, and makes an obtuse angle with the horizontal bar, that also is compressed; but if the angle between the members be acute, the horizontal bar is in tension; and, on the other hand, if the inclined bar is in tension, and makes an obtuse angle with the horizontal bar, the latter is in tension, and *vice versâ*.

A cantilever made of framed bars is shown at Fig. 28, of which the lines *a k*, *b h* are assumed to be horizontal and at a distance apart shown by *d*. In this arrangement *d* will represent the vertical height of any of the bars, and as the bars all make equal angles with the horizon, they will be of one length. The lengths are to be measured from centre to centre of the joints, and along the centre lines of the elements of the structure. Let a load W be carried at the end of the cantilever, then this load will evidently be the *load* for each of the inclined bars, for it will have to pass through all of them, entering some at the top and some at the bottom, and so bringing compression on *a b*, *c g*, *e f*, and tension on *b c*, *g e*, and *f k*; and if L be the length of an inclined bar, the strain on every inclined bar will be $= W \times \frac{L}{d}$. The horizontal projection of the bar *a b* will be equal to half of *a c*, the distance between two joints of the inclined with the horizontal bars,

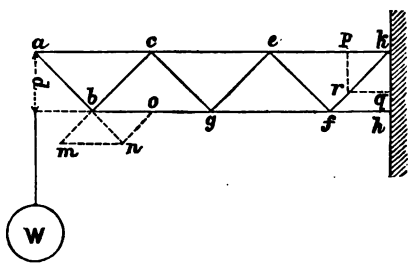

Fig. 28.

so the strain on $a\,c$ will be $W \times \frac{a\,c}{2\,d}$; and this strain will run through $a\,k$ to k.

The strain coming down $a\,b$ is at b again resolved upon $b\,c$ and $b\,g$; the strain on $b\,c$ is the same as that upon $a\,b$. Produce $c\,b$ to m and $a\,b$ to n, making $b\,m = b\,n =$ strain on $a\,b$ or $b\,c$; complete the parallelogram $b\,m\,n\,o$, then $b\,o$ or $m\,n$ will equal the strain put upon $b\,g$ by that coming down $a\,b$.

Because $a\,b\,n$ is parallel to $c\,g$, and $c\,b\,m$ is a straight line, the angle $m\,b\,n$ is equal to the angle $b\,c\,g$; and because, also, $b\,m$, $b\,n$ are equal, $m\,n$ is parallel to $b\,g$, and $b\,m\,n$ is similar to $c\,b\,g$, and the strain on $b\,g$ is to the vertical load as the distance $b\,g$ or $a\,c$ to the depth, and that strain is $= W \times \frac{b\,g}{d}$; this strain will go through $b\,h$ to h. Thus, d representing the load, there is a strain $a\,b$ on every inclined bar, and on the horizontal bars the strain is augmented at every point with an inclined bar in the following manner:—

On $a\,c$ the strain is $W \times \frac{a\,c}{2\,d}$; on $c\,e$, $W \times \left(\frac{a\,c}{2\,d} + \frac{a\,c}{d}\right)$; on $c\,k$, $W \times \left(\frac{a\,c}{2\,d} + \frac{a\,c}{d} + \frac{a\,c}{d}\right)$; on $b\,g$ the strain is $W \times \frac{a\,c}{d}$; on $g\,f$, $W \times \left(\frac{a\,c}{d} + \frac{a\,c}{d}\right)$; on $f\,h$, $W \times \left(\frac{a\,c}{d} + \frac{a\,c}{d} + \frac{a\,c}{d}\right)$; on the top flange the strains increase in arithmetical progression, as 1, 3, 5, &c.; and on the bottom flange, as 2, 4, 6, &c. The horizontal pull on the wall at k from $f\,k$ will, however, be represented by half $a\,c$, as may be shown by completing the parallelogram $k\,p\,r\,q$, and arguing upon the similarity of the triangles $k\,r\,q$, $k\,f\,h$.

The total pull upon the wall at k will be $\frac{W \times a\,c}{2\,d}(1 + 2 + 2 + 1) = \frac{3\,W.\,a\,c}{d}$.

The total thrust on the wall at $h = \frac{W \times ac}{d} (2 + 2 + 2)$ $= \frac{3\,W.\,ac}{d}$. The same results will be arrived at by taking the moments of force about h and k, and dividing by d, as in the case of the plate-webbed cantilever, for this gives $\frac{W \times ak}{d}$, and $ak = 3ac$.

Fig. 29 shows another arrangement of framed cantilever, where some of the bars are inclined and some vertical in the web. It is supposed to be loaded with a weight W at the end, L being equal to the length of an inclined bar, and d the depth equal to the length of a vertical bar.

The horizontal projection of each inclined bar is equal

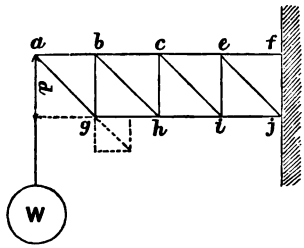

Fig. 29.

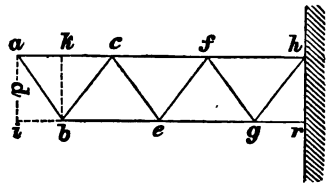

Fig. 30.

to ab, and at each joint of inclined and horizontal members the ratio of the strain brought upon the horizontal bar by the inclined to that of the inclined bar is as ab to L, and is, therefore, equal to $W \times \frac{L}{d} \times \frac{ab}{L} = \frac{W \times ab}{d}$. The strain on each inclined bar is $W \times \frac{L}{d}$, that on each vertical bar equal to W.

Let Fig. 30 represent a triangular girder loaded uni-

formly with a weight W for a length of the base of one triangle; then the loads will be at a, $\frac{W}{2}$; c and f, W; and h, $\frac{W}{2}$.

Let L = length of a lattice bar, and d = depth of girder, then the strains on the inclined bars will be, if s = strain on any bar, on $a\,b$ and $b\,c$, $-s = \frac{W}{2} \times \frac{L}{d} = \frac{W\,L}{2\,d}$; on $c\,e$ and $e\,f$, $s = \frac{W\,L}{2\,d} + \frac{W\,L}{d} = \frac{3\,W\,L}{2\,d}$; on $f\,g$, $g\,h$, $s = \frac{3\,W\,L}{2\,d} + \frac{W\,L}{d} = \frac{5\,W\,L}{2\,d}$.

Let B = base of one triangle, then the strains due to the load at a being represented by the triangle $a\,i\,b$, the resolution of strains at the intersections b, c, e, f, g, by the triangles $a\,b\,c$, $b\,c\,e$, &c.

On $a\,c$, $s = \frac{W}{2} \times \frac{B}{2\,d} = \frac{W\,B}{4\,d}$; on $c\,f$, $s = \frac{W\,B}{4\,d} + W \times \frac{B}{d} = \frac{5\,W\,B}{4\,d}$; on $f\,h$, $s = \frac{5\,W\,B}{4\,d} + \frac{3\,W\,B}{2\,d} + W \times \frac{B}{2\,d} = \frac{13\,W\,B}{4\,d}$.

We may check this by taking the moments of the loads about the point g to give the strain on $f\,h$, dividing this moment by d. The moments of the weights are for a, $\frac{W}{2} \times 2{\cdot}5\ B = \frac{5\,W\,B}{4}$; for c, $W \times 1{\cdot}5\ B = 1{\cdot}5\ W\,B$; for f, $W \times 0{\cdot}5\ B = 0{\cdot}5\ W\,B$; and the sum of these divided by $d = \frac{W\,B}{4\,d}(5 + 6 + 2) = \frac{13\,W\,B}{4\,d}$.

The process looks somewhat intricate, hence I will further explain how the increments at c and f are obtained for the horizontal member $a\,h$.

In Fig. 31 $a\,h$ is the line of the upper flange on which are to be marked the strains for subsequent adding up. First we have the horizontal strain on $a\,c$ due to $\frac{W}{2}$ at a. If $d = \frac{W}{2}$, complete (Fig. 30) the parallelogram $a\,i\,b\,k$; then $a\,k\left(=\frac{B}{2}\right)$ will be the strain on $a\,c$, or $s = \frac{W}{2} \times \frac{a\,k}{a\,i} = \frac{W}{2} \times \frac{B}{2} \times \frac{1}{d} = \frac{W\,B}{4\,d}$, and this runs through the flange to the wall, so it is marked on every bar. Now for the increment at c (Fig. 31). We have, *first*, the strain brought on the flange by the bar $b\,c$; the increment is to the strain on the bar as $n\,c$ to $c\,m$, or as B to L; hence $= \frac{W\,L}{2\,d} \times \frac{B}{L} = \frac{W\,B}{2\,d}$; *second*, the strain on $c\,f$ due to the load W on c. Make $c\,p = d = W$, then in the parallelogram $c\,p\,e\,q$, $c\,q = \frac{B}{2}$ here represents the increment due to load at $c = W \times \frac{B}{2} \times \frac{1}{d} = \frac{W\,B}{2\,d}$, which, added to the first part of the increment, makes $\frac{W\,B}{2\,d} + \frac{W\,B}{2\,d} = \frac{W\,B}{d}$, the total increment in the expression for $c\,f$, which also passes to h, as marked. As to the second part of the increment, it is to be noticed that it will be the same for f, and if there were more intersections, the same for all succeeding; but the first part is determined by the strain on the inclined bar, so we have

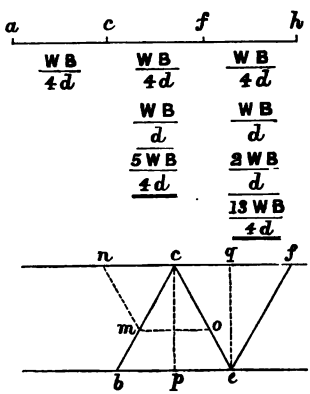

Fig. 31.

for the increment at f, *first* from bar ef, $s = \frac{3\,W\,L}{2\,d} \times \frac{B}{L} = \frac{3\,W\,B}{2\,d}$, and from the load at f, $s = \frac{W\,B}{2\,d}$, making the total increment $s = \frac{3\,W\,B}{2\,d} + \frac{W\,B}{2\,d} = \frac{2\,W\,B}{d}$. Were this to be continued as for a longer cantilever, we should find the increments increasing in value at every joint by a quantity $= \frac{W\,B}{d}$.

The strains on the bottom flange (Fig. 30) will be, on be, $s = \frac{W\,L}{2\,d} \times \frac{B}{L} = \frac{W\,B}{2\,d}$; on eg, $s = \frac{W\,B}{2\,d} + \frac{3\,W\,L}{2\,d} \times \frac{B}{L} = \frac{2\,W\,B}{d}$; on gr, $s = \frac{2\,W\,B}{d} + \frac{5\,W\,L}{2\,d} \times \frac{B}{L} = \frac{9\,W\,B}{2\,d}$.

There are no other increments, as there are no additional loads at the bottom joints.

The summing up of the strains on the lower flange can also be checked by taking the moments about the point h (Fig. 30); they will be for a, $\frac{W}{2} \times 3\,B$; for c, $W \times 2\,B$; for f, $W \times B$; total, divided by d, to give direct strain at r, $= \frac{W\,B}{d}\left(\frac{3}{2} + \frac{4}{2} + \frac{2}{2}\right) = \frac{9\,W\,B}{2\,d}$, as found above.

Let A B, Fig. 32, represent a triangular girder supported at both ends, and loaded by a weight W placed on one of

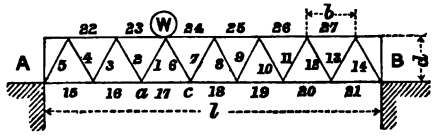

Fig. 32.

the apices of the triangles, l = the span, the remaining notation as above. By the usual method the reactions of

the supports A and B are found, being, on A, $W \times \frac{9\,b}{2\,l}$, and on B, $W \times \frac{5\,b}{2\,l}$; or as $l = 7\,b$, the reactions are, on $A = W \times \frac{9\,b}{14\,b} = \frac{9\,W}{14}$, and on $b = W \times \frac{5\,b}{14\,b} = \frac{5\,W}{14}$. Resolving these weights on the inclined bars, we shall find alternately in compression and tension on the bars 1 to 5, $s = \frac{9\,W}{14} \times \frac{L}{d} = \frac{9\,W\,L}{14\,d}$, and on the bars 6 to 14, alternately in compression and tension, $s = \frac{5\,W\,L}{14\,d}$. At every intersection the strains will be resolved as represented by the complete triangles, but the strains on bars 5 and 14 will be resolved vertically and horizontally on the supports and the bottom flange. The strains to the left side of W will be, on 15, $s = \frac{9\,W\,L}{14\,d} \times \frac{b}{2\,d} = \frac{9\,W\,b}{28\,d}$; on 16, $s = \frac{9\,W\,b}{28\,d} + \frac{9\,W\,L}{14\,d} \times \frac{b}{L} = \frac{27\,W\,b}{28\,d}$; and on 17, $s = \frac{27\,W\,b}{28\,d} + \frac{9\,W\,b}{14\,d} = \frac{45\,W\,b}{28\,d}$. On the right of W, the strain on 21 is $s = \frac{5\,W\,L}{14\,d} \times \frac{b}{2\,L} = \frac{5\,W\,b}{28\,d}$, and at every joint towards W there will be an increment $= \frac{5\,W\,l}{14\,d} \times \frac{b}{L} = \frac{5\,W\,b}{14\,d}$; so that on 17 the strain will be $s = \frac{5\,W\,b}{28\,d} + 4 \times \frac{5\,W\,b}{14\,d} = \frac{45\,W\,b}{28\,d}$, which there meets and equilibrates the strains coming from the left side of W. I will now compare this with the strain as found by taking the moments about the points under W. The strain will be equal to the moment of the reaction of A divided by d or $s = \frac{9\,W\,b}{2\,l} \times \frac{5\,B}{2\,d} = \frac{45\,W\,b^2}{4\,l\,d}$, where $l = 7\,B$, making

$s = \frac{45\ W\ b}{28\ d}$. On the top flange the increment of strain at each intersection of diagonals will be, on the left of W $= \frac{9\ W\ b}{14\ d}$, making the strain on 23, $s = 2 \times \frac{9\ W\ b}{14\ d} = \frac{9\ W\ b}{7\ d}$. On the right of W each increment will be $\frac{5\ W\ b}{14\ d}$, making the strain on 24, $s = 4 \times \frac{5\ W\ b}{14\ d} = \frac{10\ W\ b}{7\ d}$. We have therefore on the one side $\frac{9\ W\ b}{7\ d}$, on the other $\frac{10\ W\ b}{7\ d}$. Now if we consider that $\frac{9\ W}{14}$ passes down bar 1 and $\frac{5\ W}{14}$ down bar 6, we shall find that the pressure of the tops of the bars on the pin joining them will be $\frac{9\ W\ b}{28\ d}$ and $\frac{5\ W\ b}{28\ d}$ respectively, showing a difference of $\frac{4\ W\ b}{28\ d} = \frac{W\ b}{7\ d}$ pressing from the left, and therefore to be added to the strain brought upon the pin by bar 23, bringing it up to $\frac{10\ W\ b}{7\ d}$, which equilibrates the strain on bar 24.

To check the accuracy of the strain thus determined on the bar 23, the moments about the apex a may be taken and divided by d thus: $\frac{9\ W}{14\ d} \times 2\ b = \frac{9\ W\ b}{7\ d}$, and for the bar 24, about c, $\frac{5\ W}{14\ d} \times 4\ b = \frac{10\ W\ b}{7\ d}$.

The mode of procedure is of course similar if the load be on the bottom flange. If there be more than one weight on the girder the strains can be determined separately for them, and the resultants found: if on any bar all the strains are compressive, or all are tensile, the sum of strains will be the resultant strain; but if they are mixed, the difference

between the sums of the compressive and tensile strains will be the resultant strain.

In Fig. 33, let A B represent a triangular girder having a uniformly distributed load w per lineal foot on the top

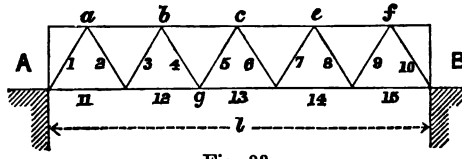

Fig. 33.

flange. It is evident that one-half the load will in this case be on each support; thus each *strut* carries all the load that *lies between it and the centre of the girder;* hence the loads, if B equal the base of one triangle on the struts, will be, on 5 and 6, $\frac{w\,\mathrm{B}}{2}$; on 3 and 8, $\frac{3\,w\,\mathrm{B}}{2}$; on 1 and 10, $\frac{5\,w\,\mathrm{B}}{2}$; and were the girder longer, the loads would continue to increase in like arithmetical progression. In order to test the accuracy of this summing of loads, I will take the load on each apex separately, and sum the results (for loads, not strains), calling tensions minus and compressions +. Load at a gives on 1, $\frac{9\,w\,\mathrm{B}}{10}$; on 2, 4, 6, 8, 10, $\frac{w\,\mathrm{B}}{10}$; but these numbers will be clearer in a tabular form, in which, however, the w B will be omitted.

TABLE OF STRAINS ON DIAGONALS.

No. of Bar.	1	2	3	4	5	6	7	8	9	10
Load on apex a	$+\frac{9}{10}$	$+\frac{1}{10}$	$-\frac{1}{10}$	$+\frac{1}{10}$	$-\frac{1}{10}$	$+\frac{1}{10}$	$-\frac{1}{10}$	$+\frac{1}{10}$	$-\frac{1}{10}$	$+\frac{1}{10}$
,, ,, b	$+\frac{7}{10}$	$-\frac{7}{10}$	$+\frac{7}{10}$	$+\frac{3}{10}$	$-\frac{3}{10}$	$+\frac{3}{10}$	$-\frac{3}{10}$	$+\frac{3}{10}$	$-\frac{3}{10}$	$+\frac{3}{10}$
,, ,, c	$+\frac{5}{10}$	$-\frac{5}{10}$	$+\frac{5}{10}$	$-\frac{5}{10}$	$+\frac{5}{10}$	$+\frac{5}{10}$	$-\frac{5}{10}$	$+\frac{5}{10}$	$-\frac{5}{10}$	$+\frac{5}{10}$
,, ,, d	$+\frac{3}{10}$	$-\frac{3}{10}$	$+\frac{3}{10}$	$-\frac{3}{10}$	$+\frac{3}{10}$	$-\frac{3}{10}$	$+\frac{3}{10}$	$+\frac{1}{10}$	$-\frac{7}{10}$	$+\frac{7}{10}$
,, ,, e	$+\frac{1}{10}$	$-\frac{1}{10}$	$+\frac{1}{10}$	$-\frac{1}{10}$	$+\frac{1}{10}$	$-\frac{1}{10}$	$+\frac{1}{10}$	$-\frac{1}{10}$	$+\frac{1}{10}$	$+\frac{9}{10}$
Resultant load	+2·5	−1·5	+1·5	−·5	+·5	+·5	−·5	+1·5	−1·5	+2·5

Each of these resultants multiplied by w B will be the *load* for the corresponding diagonal. We must now consider the strains on the flanges. The increments on the bottom flange will be for the end bars 1 and 10, the strains on those bars resolved vertically and horizontally, or to the strains on the bars, as $\frac{B}{2L}$; at the other intersections as $\frac{B}{L}$.

The strains on the diagonals will be to the loads as $\frac{L}{d}$; hence from the table are found the latter as follows:—

On 5 and 6, $s = \cdot 5\ w\ B \times \frac{L}{d} = \frac{w\ B\ L}{2\ d}$; on 4 and 7 the same but tension, as will be the case in each second pair of bars; on 3, 8, 2, 9, $s = 1 \cdot 5\ w\ B \times \frac{L}{d} = \frac{3\ w\ B\ L}{2\ d}$; on 1 and 10, $s = 2 \cdot 5\ w\ B \times \frac{L}{d} = \frac{5\ w\ B\ L}{2\ d}$.

On the bottom flange, on 11 and 15, $s = \frac{5\ w\ B\ L}{2d} \times \frac{B}{2L} = \frac{5\ w\ B^2}{4\ d}$; on 12 and 14, $s = \frac{5\ w\ B^2}{4\ d} + \frac{3\ w\ B\ L}{2\ d} \times \frac{B}{L} = \frac{11\ w\ B^2}{4\ d}$; on 13, $s = \frac{11\ w\ B^2}{4\ d} + \frac{w\ B\ L}{2\ d} \times \frac{B}{L} = \frac{13\ w\ B^2}{4\ d}$.

We will check this by taking the moments: about the apex c there is the reaction of $A = \frac{5\ w\ B}{2}$ by its leverage $\frac{5\ B}{2}$ less the downward moments of the loads at a and b, giving, when divided by d, for the strain on 13, $s = \frac{5\ w\ B}{2\ d} \times \frac{5\ B}{2} - \left(\frac{2\ w\ B^2 + w\ B^2}{d}\right) = \frac{25\ w\ B^2}{4\ d} - \frac{12\ w\ B^2}{4\ d} = \frac{13\ w\ B^2}{4\ d}$.

The top flange must now be dealt with, where the increments are in two parts, one for that brought on by the diagonal, and the other for the additional load; the process is very similar to that for the cantilever.

The portion of the increment due to the additional load will at each apex be $= w\,B \times \frac{B}{2\,d} = \frac{w\,B^2}{2\,d}$, the other part being equal to the strain on the tie multiplied by $\frac{B}{L}$. The strains will be on the top flange, on 16, $s = \frac{3\,w\,B\,L}{2\,d} \times \frac{B}{L} + \frac{w\,B^2}{2\,d} = \frac{4\,w\,B^2}{2\,d}$; on 17, $s = \frac{4\,w\,B^2}{2\,d} + \frac{w\,B\,L}{2\,d} \times \frac{B}{L} + \frac{w\,B^2}{2\,d} = \frac{6\,w\,B^2}{2\,d} = \frac{3\,w\,B^2}{d}$. This strain is checked by taking out the moments about the apex g; thus, $s = \frac{5\,w\,B}{2\,d} \times 2\,B - \left(\frac{1{\cdot}5\,w\,B^2 + {\cdot}5\,B^2}{d}\right) = \frac{5\,w\,B^2}{d} - \frac{2\,w\,B^2}{d} = \frac{3\,w\,B^2}{d}$.

It is found from the foregoing that the strains on the diagonals increase from the centre of the girder towards the points of support in an arithmetical progression, 1, 3, 5, &c., with a difference of two; so that one strain on the central triangle being calculated, the rest can be set down, and from the numbers so obtained the strains on the flanges may be found.

It is worthy of observation that in the top flange the numerical coefficient of the total increment at any intersection is the *mean of the numerical coefficients* of the strains on the intersecting diagonals; thus on 16 we find at *a*, coefficients of 1 and 2 in the table 2·5 and 1·5, of which the mean is 2, and the increment $\frac{4\,w\,B^2}{2\,d}$, showing a coefficient $= 2$, and similarly at *b e* $\frac{1{\cdot}5 + {\cdot}5}{2} = 1$.

I will now take a case in which the same girder is used, but to be loaded at the bottom instead of at the top.

Under these circumstances it will be found that the bars 5 and 6 will have no strain at all upon them, for the com-

pression on each due to the load at the foot of the other is nullified by the tension produced by the load at its own foot, therefore theoretically these bars might be abolished, but in a practical sense they are requisite to support the top flange, otherwise there would be an excessively long element in compression unsupported: the strains on the diagonals will be, on 5 and 6, $s = 0$; on 4, 7, 3, 8, $s = w\,B \times \frac{L}{d} = \frac{w\,B\,L}{d}$; on 2, 9, 1, 10, $s = \frac{w\,B\,L}{d} + \frac{w\,B\,L}{d} = \frac{2\,w\,B\,L}{d}$. Where there is not a central loaded apex it appears from this that the strains increase as 1, 2, 3, &c., with a difference of 1.

The increments of strain due to the strains on the diagonals will be determined as before, there being in this case only these on the top flange. On the bottom flange the increments due to increments of load will come, this being the only increment at g. The strains on the top flange will then be—

On $a\,b$, $s = \frac{2\,w\,B\,L}{d} \times \frac{B}{L} = \frac{2\,w\,B^2}{d}$; on $b\,c$, $s = \frac{2\,w\,B^2}{d} + \frac{w\,B\,L}{d} \times \frac{B}{L} = \frac{3\,w\,B\,L}{d}$. Checking this by the moments, $s = \frac{2\,w\,B \times 2B - w\,B \times B}{d} = \frac{3\,w\,B^2}{d}$. On the bottom flange the strains will be, on 11, $s = \frac{2\,w\,B\,L}{d} \times \frac{B}{2\,L} = \frac{w\,B^2}{d}$; on 12, $s = \frac{w\,B^2}{d} + \frac{w\,B\,L}{d} \times \frac{B}{L} + \frac{w\,B^2}{2\,d} = \frac{5\,w\,B^2}{2\,d}$; on 13, $s = \frac{5\,w\,B^2}{2\,d} + \frac{w\,B^2}{2\,d} = \frac{6\,w\,B^2}{2\,d} = \frac{3\,w\,B^2}{d}$. Checking this by the moments, we have $s = \frac{2\,w\,B \times 2{\cdot}5\,B - w\,B\,(1{\cdot}5 + 0{\cdot}5)\,B}{d} = \frac{3\,w\,B^2}{d}$.

When the girder is uniformly loaded it will be seen that all the bars pointing from the centre downwards towards the points of support are struts, and those in the reverse direction ties; but structures designed to carry such a load as this are often subjected to partial loads, which alter the nature of the strains on some of the central diagonals, and this must be attended to, in order that the bars there used may be made of sections suitable to resist compression, even though they are ties under the maximum load.

Assume that on a girder (Fig. 34) consisting of nine triangles a train has run, as shown by the larger circles, to the triangle next to that at the centre, and let the full load of girder and train be $3w$ per lineal foot, w per lineal foot being the dead load on the girder; then there will be on

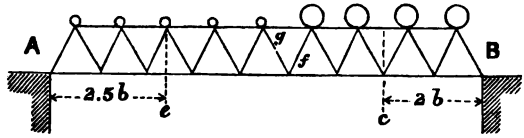

Fig. 34.

the five triangles nearest A, w B on each apex $= 5\,w$ B, of which the proportion passing towards B, and therefore in compression through the bar g, will be $5\,w\,\mathrm{B} \times \frac{2{\cdot}5\,\mathrm{B}}{l}$ $= \frac{12{\cdot}5\,w\,\mathrm{B}^2}{l}$. On the four triangles nearest B, the load on each apex is $3\,w\,\mathrm{B} = 12\,w\,\mathrm{B}$ in all, of which the part passing towards A, and therefore putting the bar g in tension, is $12\,w\,\mathrm{B} \times \frac{2\,\mathrm{B}}{l} = \frac{24\,w\,\mathrm{B}^2}{l}$, so that the tensile strain is the greater, and under this position of the load the bar g will be a tie, and the bar f will be a strut; just the reverse of their conditions under the maximum load.

I have so fully worked the details of the examples given

in order to avoid the chance of a doubt in the student's mind as to the mode of procedure, and I do not think it necessary to take further examples in illustration.

If there is more than one series of triangles crossing each other, then, of course, the strains on the bars are proportionately divided. The most ordinary angles used in practice for the diagonals are 60 degrees to the horizon, when $\frac{L}{d} = 1{\cdot}154$, and $\frac{B}{d} = 1{\cdot}154$, and 45 degrees to the horizon, when $\frac{L}{d} = 1{\cdot}414$, and $\frac{B}{d} = 2$.

The forms in which braced structures may appear are endless in their variety, and each new arrangement will probably call for some corresponding alteration of detail in determining the strains; but as the centre lines of the elements form bases for the parallelograms of forces, there cannot be much difficulty in working any form out when the *rationale* of the method has been once fairly grasped, although it must be said that diligent care must be used to insure against missing anything.

There are, however, certain typical forms which it will be desirable here to examine, of which I shall now take that shown at Fig. 35, which is known as a bowstring girder. A is one of the supports, and the load is carried on the bottom member, or the string of the bow; the loads on the different joints being indicated by w', w'', w'''. The bow is formed of straight elements, or if, for appearance' sake, it is curved, a line drawn in any bay from the centre of one joint with a diagonal, to that of the other end joint, must keep well within the depth of the element, otherwise it will be subject to an undue bending stress.

This structure, which is purely a braced girder, must not (because to the uneducated eye it bears some resemblance to it) be confounded with the *tied arch*, where the upper curved member is a true arch, having its abutments held

together by a tie, and in which the diagonal counterbracing (as will be shown subsequently) carries no part of the load, but serves to support the tie and distribute the load

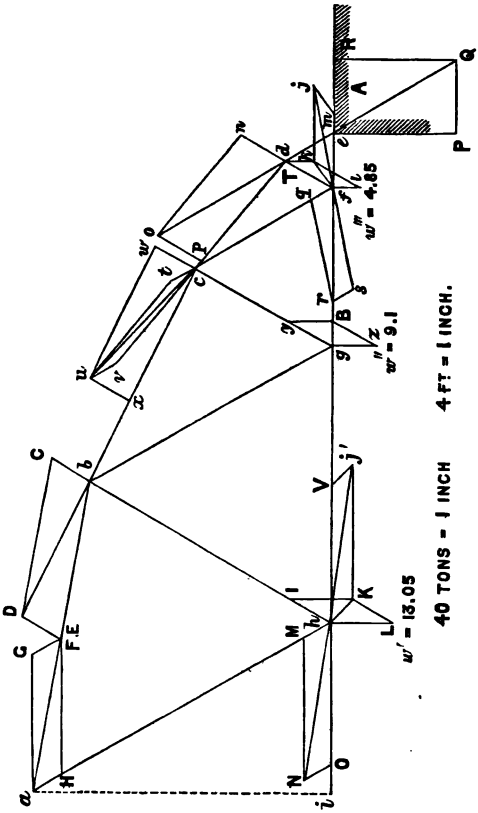

Fig. 35.

when unequal, so as to prevent excessive distortion of the arch.

The diagram of the half-girder is drawn to a scale of four feet to one inch, and for the parallelograms of forces

the scale taken is forty tons to one inch. In actual practice a much larger scale would be taken, but we are here limited to space, and even at this scale very close approximations to accuracy may be obtained by using a well-divided diagonal scale.

The top member of the girder comprises the bars *a b*, *b c*, *c d*, and *d e*; the bottom member, *e f*, *f g*, *g h*, *h i*; the diagonals are, *a h*, *h b*, *b g*, *g c*, *c f*, and *f k*.

A load of two tons per lineal foot is supposed to be on the bottom member of the girder, so it may be considered as aggregated at the points *h*, *g*, and *f*, where it amounts to $w' = 13{\cdot}05$ tons; $w'' = 9{\cdot}1$ tons; $w''' = 4{\cdot}85$ tons. These three added together will = 27 tons, which is the load on and the amount of upward reaction of the support A, which, acting vertically at the point *e*, is to be in the first place resolved into strains upon bars *e d* and *e f*. Make *e* P = 27 tons, complete the parallelogram *e* P Q R, join *e* Q, then *e* R will equal the tension on *e f*, and *e* Q the compression on *e d*. Produce *e d* to *o*, making *d o* = *e* Q; this strain must be carried by the bars *d c*, *d f*; produce *f d*, and complete the parallelogram *d p o n*: *d p* will be the strain on *d c*, and *d n* that upon *d f*—the former compression, the latter tension. On *f d* mark off *f* T = *d n*, and from *f* draw the vertical line *f l* = w''' = 4·85 tons; complete parallelogram *f* T *k l*, then *f k* is the resultant of the strain on *f d*, and the weight w'''.

From *f* mark off *f m* = *e* R, and complete the parallelogram *f k j m*; *f j* will be the new resultant, which must be carried by the bars *f g*, *f c*—tension on the former, and compression on the latter; produce *j f* to *s*, making *f s* = *f j*, and complete the parallelogram *r s f q*; *f r* is the strain on *f g*, and *f q* that on *f c*. Again, produce *d c* to *v*, making *c v* = *d p*, and *f c* to *t*, making *c t* = *f q*; complete the parallelogram *c t v w*, then *c u* is the resultant of the strains on *d c* and *f c*, and is sustained by the bars *c b*

and $c\,g$. Produce $g\,c$, and complete the parallelogram $c\,x\,u\,w$; then $c\,x$ is the compression on $c\,b$, and $c\,w$ the tension on $c\,g$. Make $g\,y = c\,w$, and from g draw vertically $g\,z = w'' = 9{\cdot}1$ tons; complete the parallelogram $g\,y\,\mathrm{B}\,z$; $g\,\mathrm{B}$ will be the new resultant, which happening to fall on the line of $g\,h$, will be borne entirely by that bar, putting no strain on $g\,b$; the total strain on $g\,h = fr + g\,\mathrm{B}$. Again, produce $c\,b$ to D, making $b\,\mathrm{D} = c\,x$; produce $h\,b$, and complete the parallelogram $b\,\mathrm{E\,D\,C}$, then $b\,\mathrm{E}$ will be the strain on $b\,a$ in compression, and $b\,c$ the tension on $b\,h$; make $h\,\mathrm{I} = b\,\mathrm{C}$, and draw $h\,\mathrm{L}$ vertically equal to $w' = 13{\cdot}05$ tons; complete the parallelogram $h\,\mathrm{L\,K\,I}$, then $h\,\mathrm{K}$ will be the resultant of the strain on $h\,b$, and the weight w'. Make $h\,\mathrm{V} = fr + g\,\mathrm{B}$, and complete the parallelogram $h\,\mathrm{K\,J\,V}$; $\mathrm{J}\,h$ will be the new resultant, to be carried by the bars $h\,i$ and $h\,a$. Produce $\mathrm{J}\,h$ to N, making $h\,\mathrm{N} = \mathrm{J}\,h$, and complete the parallelogram $\mathrm{O\,N\,M}\,h$; $h\,\mathrm{O}$ is the tension on $h\,i$, and $h\,\mathrm{M}$ the tension on $h\,a$. From a mark off $a\,\mathrm{F} = b\,\mathrm{E}$, and complete the parallelogram $\mathrm{H\,F\,G}\,a$, making $\mathrm{H\,F}$ horizontal; then, if the diagram has been accurately worked out, $\mathrm{H\,F} = h\,\mathrm{O}$, and $a\,\mathrm{H} = \mathrm{M}\,h$. That being so, let us check our results by the system of moments. The reaction at $e = 27$ tons; the intermediate loads are distant from i as follows:—w' at 3·6 ft., w'' at 9·45 ft., and w''' at 12·7 ft. The length $e\,i$ is 13·85 ft., and the depth $i\,a$ is 6·24 ft. The strain on $h\,i$ will therefore be—

$$\frac{27 \times 13{\cdot}85 - (13{\cdot}05 \times 3{\cdot}6 + 9{\cdot}1 \times 9{\cdot}45 + 4{\cdot}85 \times 12{\cdot}7)}{6{\cdot}24}$$

$= 28{\cdot}74$ tons. The line $h\,\mathrm{O}$ scales 29·1 tons, showing a difference of 0·36 for errors in drawing the parallelograms of forces, not much over 1 per cent. of the total.

The diagram being completed, the strains on the various bars can be scaled off, and the quantity of material required for each determined.

In some cases, as for roofs, the load will be upon the

top member, but the method of determining the strains will be analogous to that illustrated in this example.

The crescent girder is a structure similar in principle to the above, and calculated in the same way, but it has the bottom or tension member considerably curved, so as to give the whole rib the appearance of an arch pointed at each end. This form is much used for roofs, having a very elegant appearance, and being, moreover, an economical form of rib. The ends of all these girders must be left free for expansion and contraction to take place, otherwise, under changes of temperature, they will be subject to undue strains, and in expanding may even bend and become permanently crippled, and, moreover, if firmly fixed at the ends, the tie must buckle, as it cannot extend under the tension thrown on it; in fact, by fixing the ends, the whole equilibrium of the structure is destroyed. I refer to this, as cases have occurred of the ends of a crescent girder being firmly screwed down, with the result of distorting the girder laterally, until, upon having its feet released, it assumed its normal shape.

I have now shown the method of determining the strains on braced structures by sections, as in the lattice girder, and by reactions, as in the example last considered; but there is another system of calculation by dividing the structure into primary, secondary, tertiary, &c., trusses, which is generally employed in determining the strains upon ordinary roof trusses, and these I shall now proceed to investigate.

In Fig. 36 A B C represents the commonest shape of truss for roofs of small span. C A, C B are called the rafters, C *f* the king rod, and A B the tie. Although in practice the rafters are in one piece, also the tie, I have drawn them as separate bars laid together, in order to show how the trusses are calculated. A B C is the primary, and A *d f*, B *e f* are secondary trusses; others coming within these would be tertiary trusses, and so forth.

The load upon the roof will be regarded as concentrated at the points C, *d*, *e*, A, B. The weight at *d* is firstly supposed to come upon the secondary truss A *df*. Draw the vertical line *d g* equal the load at *d*; complete the parallelogram *d h g i*, then *d h* and *d i* will represent the strains on *d n* and *df*. From *n* mark off *n k* equal to *d h*; this strain must be resolved vertically on the point of support, and upon the

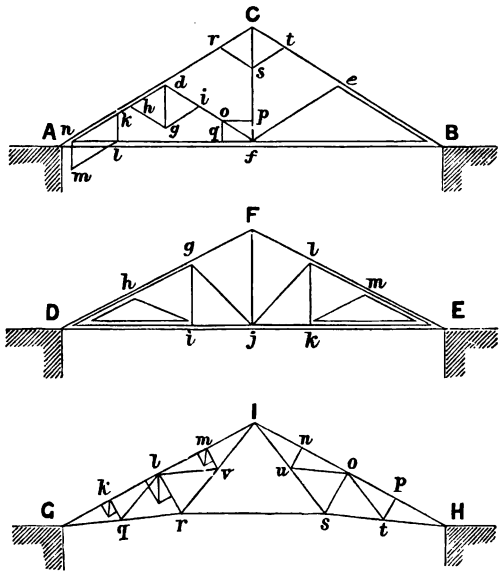

Fig. 36.

bar *nf*. Complete the parallelogram *n k l m*, then *n m* is the vertical component, and *n l* the strain on the bar *nf*. From *f* mark off *f o* = *d i*; this strain must be resolved between the king rod *cf* and the tie *fn*. Complete the parallelogram *q o p f*, then *fp* = strain on *f*C, and *fq* = the strain on the tie *fn*. Upon the king rod there will also be another strain brought upon it by the other secondary

truss *f e* B; let C *s* represent the sum of these strains; they must be resolved on the rafters C A and C B. Complete the parallelogram C *r s t*, then C *r* is the strain on C A, and C *t* that on C B. The strain C *r* must be marked off from A towards C, and resolved vertically and on the tie A B. As the rafter is made in one piece, the strain upon it will be C *r* between C and *d*, and between *d* and A it will be C *r* + *d h*; and in like manner the strain on the tie will be the sum of the strains on the ties *n q* and A B.

At D E F is shown another form of roof truss, where there are tertiary trusses *h* D *i*, *m* E *k*, in secondary trusses *g* D *j*, *l* E *j*, the whole resting in the primary truss D E F; *g i*, *l k* are queen rods. In this arrangement the load at *h* is first taken, and resolved on *h* D, *h i*; then the load on *g*, with that coming up, *g i*, from the end of D *h i*, and finally the load at F, with the strain on F *j* brought up from the ends of the two secondary trusses *g* D *j*, *l* E *j*. After all the strains have been determined, they are added together as before.

G H I is another form of roof of light design, but not so steady as those preceding it. It consists of two primary trusses, G *r* I, H *s* I, leaned against each other at I, and tied below by the bar *r s*.

In the first place, the loads at *k l m* are marked off on vertical lines, and parallelograms are completed as shown to determine the strains resulting from these loads on *m l* and *m v*, *l k* and *l r*, and *k* G and *k q*. The strains on *m v* and *h q* will again have to be resolved, the former on *v* I and *v l*, the latter on *q l* and *q* G. The strains on *v l* and *q l* will produce a resultant strain on *l r*, which, together with the strain brought upon it by the load at *l*, will have again to be resolved on the bars *r* I and *r* G; the strain on *r* I being resolved at I, normally to and on I G, and the strain on *r* G being resolved on G I and at right angles to it. The total thrust must be resolved horizontally and vertically to find the tension on *r s*.

The roofs carried by arched ribs and girders are of course calculated in the same way as the same elements when used for other purposes.

In roof trusses, where the loads are actually distributed continuously along the rafters, it is evident that these elements undergo transverse in addition to longitudinal strain; hence care must be taken that they be made sufficiently rigid to withstand such bending strain. From this effect of the load it will be found that in small roofs an apparently great excess of material must be employed in order to secure rigidity.

In all the braced structures and arrangements of frames discussed in this chapter, the bars in them are all parts belonging properly to such structures—that is to say, being necessary to them to maintain their equilibrium; but in complicated works there occur systems of bracing which do not belong to any particular girder or member, so far as in helping it to resist the strains proper for it to bear, but which serve to connect the various girders and to protect them against the action of external forces, against which their own construction does not provide.

As bracings such as these appear to be quite distinct in their functions from those combinations which I have described, and of which I have investigated the principles above, I shall discuss them and their modes of application in a separate chapter.

CHAPTER V.

ADVENTITIOUS BRACING.

This bracing is that which is added for the purpose of securing solidity and rigidity in a structure, and is generally applied to resist external causes of vibration, or, if such vibration must occur, to cause the structure to act as a whole, not to be shaken about in detail, so that the solid mass of the *whole structure* may be brought into action when *any part* of it is attacked by a vibratory force.

This bracing, sometimes called counterbracing, occurs in all kinds of positions—vertical, horizontal, and diagonal—and is of two kinds, plate bracing and bar bracing, the former being used in the form of gussets to preserve the angular positions of parts of a structure in relation to each other; but we have here more particularly to deal with bar bracing.

In Fig. 37, *a b* and *c d* represent a pair of bracing bars intended to maintain the proper form in the rectangular frame to which they are attached. These bars may be simply riveted on, or they may be attached under what is called *initial tension*, and this I propose to discuss before going further. If the work is put together merely as a fit —that is, no strain is on any part of it by reason of process of manufacture—let us examine the action when a disturbing strain comes into play. Let a force P act at *c*, in the direction shown by the arrow; this will tend to open the

angles *a c b*, *a d b*, and diminish the angles *d a c*, *d b c*, to compress *c d* and elongate *a b*. The letters *a b c d* are to be taken as placed at the centres of the pins connecting the bracing bars with the rectangular frame. In order that the bars may act properly they must not be riveted together at their intersection *e*, for if they are so riveted any disturbing force will bring transverse strain upon them. It is very important that bracing should act as soon as possible, that it should not allow much distortion to take place before its effect is felt in opposing it. To get this quick action initial tension has been applied; that is, the bars are put on, and so adjusted that when the structure is undisturbed they all have a certain amount of tension on them. At first sight this seems like getting a good hold of the work, but let us see what actually happens. Suppose the initial tension to be one ton per sectional inch of bar, directly distortion commences one bar (the one being lengthened by such distortion) is already acting with this amount of force to resist it; but on the other hand the other bar (that that would be shortened by the distortion) is by the same amount assisting at the distortion, so that so far nothing is gained in resistance to distortion by the use of initial strain, and therefore it is an unnecessary addition to the stresses to which the bars will in the ordinary course of affairs be subject. If it is proposed to use initial strain to make sure that the bracing bars are not loose, then the point of bad workmanship is touched upon, and if that be admitted it is as likely to occur in one case as in another; then if the initial tension is carelessly applied there may

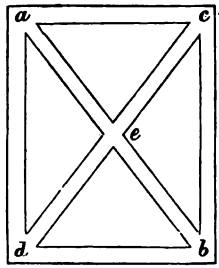

Fig. 37.

be more of it on one bar than on another, so that under such circumstances we start with a tendency (which will be *always* in action) to become distorted actually resident in the work itself, and this is certainly about as bad a state of affairs as can be imagined.

In order to ascertain the action of the bracing bars, we must observe if the lengthening and shortening efforts are the same under distortion, for if not then we may find a good reason for using initial strain.

If (in Fig. 38) the frame *a b c d* be distorted to the position shown by *a′ b′ c d*, it will be found that the shortening of the line *b c* will exceed the lengthening of the line *a d*; and that it should be the case is evident, for if we suppose the frame *a b c d* to collapse entirely, so that the point *b* lies upon *c*, and the lines *a b*, *b d* shut up on *a c*, *c d*, it will be observed that the point *b* will have travelled through the length of a diagonal *b c*, while the point *a* has travelled through a distance equal only to the difference between the length of a diagonal and that of two sides. If, for example, the frame were square, calling each side equal to 1, then the diagonal will be equal to 1·414, so the distance travelled by *b* will be 1·414, while that passed through by *a* will only amount to 2 — 1·414 = 0·586.

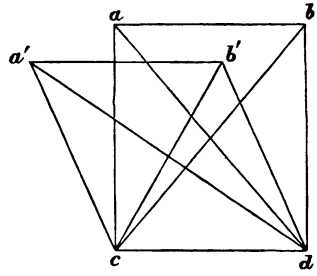

Fig. 38.

If, then, initial strain be applied, the force assisting distortion will diminish very rapidly: considering that in this case ties are also practically more effective than struts, it seems probable that the judicious use of initial strain in some classes of counterbracing may be advantageous.

Another point may also be noticed, which is, that with-

out initial tension, the bar under compression rapidly becomes more strained than that under tension, whereas in the other case it first loses its initial tension, and then starts to overtake the amount of strain on the tie, so that the ultimate stresses may be more equal.

In Fig. 39, let *a b h g* represent a pier consisting of two columns braced together as shown. If there were merely superincumbent weight to be sustained, no adventitious bracing would be requisite; but there are vibration and the wind to be withstood, and that force of the wind will be not only on the pier itself, but also on the superstructure, whatever it may be. If the wind be assumed to be blowing in the direction indicated by the arrow, its effect will be to take a part of the weight off the column *b h*, and put it on *a g*; that is, assuming the pier to be solidly braced together. Let H = height of pier, and B = breadth of base from centre to centre of columns, W = load virtually removed from *h* to *g*, P total pressure of the wind on the pier, including that on the superstructure. Let H′ = the height to the centre of pressure of the wind, then the moment of its force about *h* will be P × H′, and the extra pressure or load upon *g* will be $W = \frac{P.\ H'}{B}$, and this must all pass through the bracing bars. I shall consider the bars as attached without initial strain.

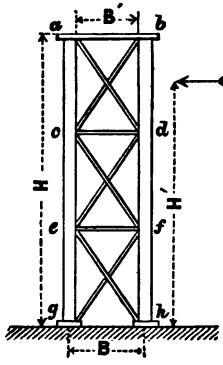

Fig. 39.

The load W is virtually suspended at the points *b*, *d*, *f*, and *h*, one-sixth of it being taken by each of the diagonal bracing bars. There will then be a compression on *b c*, *d e*, *f g*, and *g h*, and tension on *a b*, *a d*, *c f*, and *e h*. If the height

of the pier is equally divided by the bays of bracing there will be no strain on *c d* and *e f*, because the tension of one diagonal is equilibrated by the compression of the following one, for if the pier is sufficiently rigid, we may assume the tensions and compressions on the diagonals as equal. Let L = length of diagonal.

The amount of strain on each diagonal will be $\frac{W}{6} \times \frac{L}{\frac{1}{3} H} = \frac{W L}{2 H} = \frac{P. H'. L}{2. B. H}$. Calling B′ the distance between the pins of the diagonals measured horizontally, the strains acting horizontally will be $\frac{W}{6} \times \frac{B'}{\frac{1}{3} H} = \frac{P. H'. B'}{2 B H}$, and if the diagonals are not carried where their ends meet by the same bolt, there will be this amount of shearing strain on the column, or on the bolts connecting the columns if a joint occurs at that place, as is commonly the case.

I will take an example for the sake of illustration. Let the height of the pier be 30 feet, the distance between the centres of the columns 8 feet, the diameter of each column 18 inches, and the pressure of the wind 40 lbs. per superficial foot, which is about what should be taken for calculation in England.

Let the exposed surface of the superstructure be a plate girder, of which each pier carries 40 feet in length, being 5 feet deep. As to the exposed surface only one plate girder will require to be taken as being so close together that one will shelter the other; if lattice girders were used, the area of both would be taken, as the wind will blow through one upon the other, and for a similar reason it will be taken as blowing upon both the columns: this will be equal to the force on one *flat* surface 18 inches wide, round bodies offering but half the resistance of flat surfaces. The pressure on the girder will then be 40 × 5 × 40 = 8,000 lbs. at a mean height of 32·5 feet, giving a moment 32·5 ×

8,000 = 260,000 foot lbs. The moment of the wind pressure on the columns will be, $30 \times 1{\cdot}5 \times 40 = 1{,}800$ lbs. multiplied by the mean height 15 feet, $1{,}800 \times 15 =$ 27,000 ft. lbs., making the value of $W \frac{260000 + 27000}{8 \times 2240}$ = 16 tons, of which one-sixth goes on each diagonal, and $\frac{W}{6} = \frac{16}{6} = 2{\cdot}66$ tons. The length of each diagonal will be, taking $B' = 6$ feet, 11·66 feet; hence the strain on each diagonal $= 2{\cdot}66 \times \frac{11{\cdot}66}{10} = 3{\cdot}108$ tons. If there are more tiers of columns besides the others, so as to make three rows, the wind strain on the pier will be greater by the additional columns, but it will also be further distributed through the additional bracing. Although in a small structure, like that taken as an example, the strain does not amount to much, yet it rapidly increases with the height of the work.

The horizontal strain is $2{\cdot}66 \times \frac{6}{10} = 1{\cdot}6$ tons, tending to slide the columns on the joints. If a bridge be carried on piers so as to be fixed on some, and loose or on rollers on others (to allow for expansion and contraction), and suddenly the brakes be applied on a passing train, the retarding force must act upon the piers to which the girders are fixed, and tend to rock them longitudinally; it is advisable to consider this strain in addition to that caused by the wind in arranging the bracing of groups of columns for bridge piers. The amount of the force will depend on the efficiency of the brakes, but the maximum possible force will occur when the wheels are suddenly skidded throughout the train. Let the weight of the train be taken at 200 tons, and the coefficient of friction of the wheels on the rails at 0·12, then the friction of the whole train with all the wheels skidded will be $200 \times 0{\cdot}12 = 24$ tons. So that if there

were in the grouping two columns (as Fig. 39) under each line of the rails, the strain on the bracing would be 12 tons per pair, and this multiplied out as before gives, taking the force as acting at the top of the pier, $W = \frac{12 \times 30}{8}$ $= 45$ tons. Then the strain on each diagonal $= \frac{W L}{2 H} = \frac{45 \times 11{\cdot}66}{2 \times 30} = 8{\cdot}745$ tons, and each horizontal strain $= \frac{W B'}{2 H} = \frac{45 \times 6}{2 \times 30} = 4{\cdot}5$ tons.

The force in any case will be equal to the weight skidded by the coefficient of friction, but if the wheels be not skidded, then the force will be the total pressure put on the brakes multiplied by the coefficient of friction between the brake blocks and the wheels. Here the surfaces of contact give a better hold than the peripheries of the wheels on the rails, and for metal brake blocks the coefficient of friction will be 0·25, and for timber brake blocks 0·5.

The floor or horizontal bracing does not readily admit of calculation for vibration due to running loads, but it does when the bridge is regarded as a lattice girder laid on its side to resist the lateral force of the wind blowing on one side girder, when each girder plays the part of a flange in resisting this force.

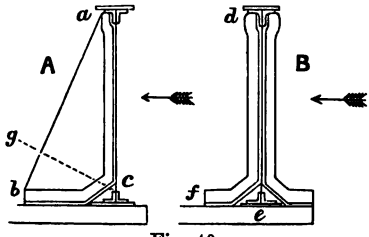

Fig. 40.

I must now say a few words about solid bracing, or solid attachments that act in the place of bracing, such as gusset plates and stiffeners. In Fig. 40, at A is shown the attachment of a main girder

in cross section to a cross girder, stiffened by means of a gusset plate *a b c*.

Here the gusset plate acts like one half of a beam. The wind pressure tending to overturn the girder acting in the direction of the arrow will bring the maximum bending strain upon the gusset plate at or about the line of section *g c*, the strength of which may be calculated in the ordinary way; but the point *c* is fixed as a centre of moments, and there fixes the neutral axis. At B is shown a stiffening arrangement, consisting of a tee iron only, riveted to the web of the girder, and turned round and riveted on to the top of the cross girder. As this to the eye does not look very stiff, I will take a practical case, and calculate the resistance offered to the wind. The girder is 8 feet deep, and stiffened by tee irons 6 inches on the back, 4 inches on the web, and ¾ inch thick, placed 4 feet apart; the section being shown in Fig. 41. Regarding the section as made up of the central part *e e*, and the two side pieces *c c*, and taking the direct working resistance as the Board of Trade allows, 5 tons per sectional square inch in compression and tension, the moment of resistance about *a a*, as a neutral axis, will be, by the formula given in a previous chapter,

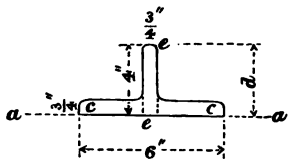

Fig. 41.

$$M = \frac{s}{3\,d}(b\,d^3 + b\,d'^3) = \frac{5}{3 \times 4}(\tfrac{3}{4} \times 4^3 + 5{\cdot}25 \times (\tfrac{3}{4})^3) =$$

20·8 inch tons for each tee iron, or 41·6 for the two. Then dividing by the height of the centre of wind pressure 48 inches, and by the area carried by each pair of stiffeners, 8 feet × 4 feet, and reducing to lbs., the effective working resistance of the pair will be $\frac{41{\cdot}6 \times 2240}{48 \times 8 \times 4} = 60{\cdot}7$ lbs. per superficial foot. And at the working pressure adopted by

many practical engineers of 3·5 tons for compression, and 4·5 tons for tension, the resistance will be 48·5 lbs. per superficial foot, an ample allowance. The last kind of adventitious bracing to which I shall here have to refer is the counterbracing used in tied arches, and although as such it comes properly in this place, the student may miss it until he has mastered the theory of the arch, which is dealt with in a subsequent chapter.

Let Fig. 42 represent a tied arch carried on supports A B, A *e* B being the arch, A *o* B the tie holding the abutments in position, and the load being on the level of the tie, and carried by the suspenders *a j*, *b k*, *c m*, &c. The

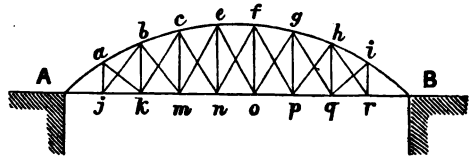

Fig. 42.

counterbracing is formed by the diagonal bars *a k*, *b j*, *b m*, *c k*, &c., the object in adding these to the structure being to aid in distributing partial loads so as to avoid distortion of the arch. We find here, as in all other cases of adventitious bracing, that its duty is to resist the forces causing distortion. Suppose a concentrated load to be at *m* carried up to *c*, its tendency will be to pull down and flatten that part of the arch about *b* to *e*, throwing up the part from *f* to *h*; but the elevation of *f* will immediately bring a tensile force on *f n*, which, acting upward through *n e*, will tend to support the point *e*, and from there it will pass on to take part of the load off *m* up the bar *m e*; the bars being all ties, *e o* and *c n* are not brought into action.

If in an arrangement of this description initial tension is put on the diagonals, there must be initial compression on all the uprights, and the result of this will be that when the weight comes on the structure the first part of it will

be taken up by the diagonals, until they are sufficiently elongated to let out the compression on the uprights, which will then come in to share in carrying the load. The amount of strain that can come upon the counterbracing can be calculated from the maximum load coming on the bottoms of the bars.

In regard to bracing generally, it is very evident that great care should be taken to have the bars of correct length, for if they are not properly proportioned they will be useless, and perhaps worse, as leading to a feeling of security, and thus causing strains to be fearlessly put upon works quite inadequate to sustain them.

It is the more necessary to impress on the mind of the reader the necessity of giving particular attention to the bracing described in this chapter, because there exists an inclination in some to regard it as a matter of guess-work (a matter of practical experience they will perhaps call it), and put in bars without obtaining any data, or making any calculations as to the proper quantities of material in use.

The arrangement of the bracing in relation to the other elements of the structure must also be considered early in the progress of the general design. We must not go on designing the girders and main load-carrying elements in the faith that the bracing can afterwards be got in somehow, or very likely we may find there is not room enough or convenience of form to make suitable and workmanlike connections between the bracing and the principal girders, and a patchy joint is only less objectionable than one joint that requires another to carry it, or connect it with the main girders; this latter contrivance is indisputable evidence of incompetence or gross carelessness on the part of the designer.

I have thought it necessary here to impress the importance of the proper disposition of these joints, but the

methods of forming these will be fully entered upon in the subsequent chapter devoted to joints of all descriptions.

Where the bracing bars are so long that they will be liable to bend by their own weight, they may be stiffened by riveting or bolting together at the points of intersection; but in short bars this is not necessary, and should be avoided for reasons previously given.

Long bars acting as struts must be regarded as columns, and calculated accordingly, and not taken as exerting the full resistance of their sectional areæ to the crushing force.

And, finally, all bars used for bracing purposes should be rigorously examined in order to be sure that they are *perfectly straight*, which is a point of paramount importance.

CHAPTER VI.

DEFLECTION AND DISTORTION.

THE determination of the deflection of a girder is not a matter that is very satisfactorily solved by calculation, by reason of the variations of the modulus of elasticity, for this does not strictly follow the law generally accepted, and is not constant for all intensities of strain.

It was found in a series of experiments on the elasticity of cast-iron bars 1 inch square and 10 feet long, under tensile strains ranging from 0·47 to 6·6 tons, the modulus of elasticity ranged from 14,050,320 lbs. to 9,549,120 lbs.; there was permanent set from the second load 0·7 ton, so the elasticity appeared to be injured at as light a strain as that, and keeping within the bounds of the extreme load permitted in practice, we find with a load of 2·82 tons per square inch a modulus of 13,166,300 lbs., the modulus decreasing rapidly with an increase of strain. In compression the modulus of elasticity was at 0·92 ton, 13,214,400 lbs., and at 16·56 tons, 10,836,480 lbs.

In twenty experiments on wrought-iron bars the modulus of elasticity varied from 29,119,800 lbs. at 0·56 ton per square inch, to 26,335,080 lbs. at 11·26 tons.

From these figures it is evident that not only does the modulus vary with the load, but also that it is not the same for tension and compression.

In Fig. 43, let A B represent a portion of a bent beam,

the dotted line showing the neutral axis, and O being the centre of curvature from a; draw $a\,f$ parallel to $b\,e$, then $e\,f$ is the extension of the fibre $e\,c$. By similar triangles O a is to $a\,b$ as $a\,e$ is to $e\,f$. Let $\frac{d}{2}$ = depth $e\,a$, E = modulus of elasticity, s = strain per square inch on fibre $e\,c$, and R = radius of curvature at e.

$$\frac{\mathrm{O}\,a}{a\,b}=\frac{a\,e}{e\,f} \quad \therefore \quad \frac{\mathrm{R}}{1}=\frac{\frac{d}{2}}{\frac{s}{\mathrm{E}}} \text{ and } \mathrm{R}=\frac{d\,\mathrm{E}}{2\,\mathrm{S}}.$$

If the strain be uniform throughout the length of the girder, then the radius of curvature is constant, and the deflection at any point is obtained from the ordinary rule thus:—Let $e\,g$ be a tangent to the circle at e, then to find $g\,h$ we have—

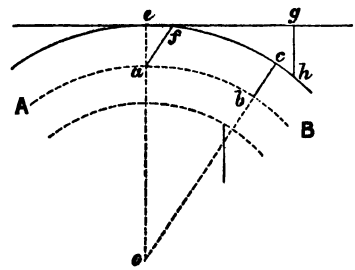

Fig. 43.

$$g\,h=\frac{e\,g^2}{2\,\mathrm{R}},$$

which is accurate to within a very trifling error not noticeable in practice. The proof of this will be found in any elementary work treating of the properties of the circle.

Applying this to girders having uniform strain throughout, where l = span of girder, and d = depth, c strain on top flange, and t = do. on bottom flange, $e\,g=\frac{l}{2}$, $g\,h=$ D = deflection at centre. Supposing the girder to be supported at the ends,

$$\mathrm{D}=\frac{e\,g^2}{2\,\mathrm{R}}=\frac{\frac{l^2}{4}}{2\,\mathrm{R}}; \qquad \mathrm{D}=\frac{\frac{l^2}{4}}{\frac{2\,d\,\mathrm{E}}{c+t}}=\frac{l^2\,(c+t)}{8\,d\,\mathrm{E}}.$$

Here $c + t = 2s$; and similarly for a cantilever, $D = \frac{l^2 (c + t)}{2. d. E.}$.

In practice it is seldom such a condition can be found, and if the sectional area and moment of resistance vary, so will the radius of curvature; so it is advisable to see in what way the deflection varies, and then to fill in constants from experiment.

Returning to the first equation, $R = \frac{d E}{2 S}$, and including all terms except the load and known dimensions in the constant a, to be determined by experiment, we find—

$R = a \frac{d}{s}$, $D = \frac{l^2}{8 R} = \frac{l^2}{\frac{8 a d}{s}} = \frac{l^2 s}{8 a d} = a' \frac{l^2 s}{d}$; but s at the centre *varies as* $\frac{W l}{b d^2}$ for solid rectangular beams, or $= a'' \frac{W l}{b d^2}$. $D = a' \frac{l^2}{d} \times a'' \frac{W l}{b d^2} = a''' \frac{W l^3}{b d^3}$, in which the value of a''' must be filled in from experiment.

Where the section of the beam is not a *solid* rectangle, $b d^3$ is to be replaced by $b d^3$, $b' d'^3$ . . . $b^n d^{n3}$, as in the determination of moments of resistance; let this quality $= m$.

For iron of good quality, we find the following formulæ by filling in the value of a''' from experiments:—l = span in feet, W = load in tons, b and d = breadth and depth in inches, D = deflection in inches.

Cast Iron.—Girder loaded at the centre, $D = \frac{W. l^3}{14 m}$; loaded uniformly over its length, $D = \frac{W l^3}{22{\cdot}4\ m}$. Cantilever loaded at the free end, $D = \frac{8. W l^3}{7. m}$; loaded uniformly over its length, $D = \frac{3 W l^3}{7. m.}$

WROUGHT IRON.—Girder loaded at the centre, $D = \frac{W l^3}{28. m}$; loaded uniformly over its length, $D = \frac{W l^3}{44{\cdot}8. m.}$. Cantilever loaded at the free end, $D = \frac{4 W l^3}{7 m}$; loaded uniformly over its length, $D = \frac{3 W l^3}{14. m.}$.

This is for sound wrought iron, such as square bars or sound forgings. The rolled iron shows much greater deflection when beam and H sections are reached, and this may be due to the fact that for complex sections the iron must be softer to enable it to follow the form of the rolls. For rolled girder we find—

Central load, $D = \frac{W l^3}{18 m}$; uniformly distributed load, $D = \frac{W. l^3}{29. m.}$.

Very great variations may be expected in riveted work; but the following expressions are taken from samples of good ordinary work, taken within working limits, the girders having the plates distributed so as to approximate as nearly as possible to uniform strain per sectional square inch, the strains varying from 3 to 5 tons per square inch. Single web plate girder, loaded at the centre, $D = \frac{W l^3}{16. m}$; loaded uniformly, $D = \frac{W l^3}{25. 6. m.}$.

The double web plate girders come up to the rolled beams given above.

Framed structures will distort under strain, and in fact a lattice girder may be said to deflect as a whole, though none of its elements are under bending stress; from the shortening and lengthening of the bars composing the triangles the distortion may be determined.

As I have observed above, the formulæ for deflection are

not practically of much use, for although the constants may be determined for bars, yet they cannot be made to include all the varied conditions of material and manufacture; they are therefore chiefly of use for purposes of comparison, and to enable us to determine the requisite proportions of combined girders in a structure, for if one sort of iron is being used throughout a structure, or a series of structures, the coefficient of deflection, whatever it is, should be the same for all; that is to say, its variation should not exceed the limits of variation for differences of strain, and as presumably all parts of the work will be calculated to undergo the same strain per square inch, the coefficient of deflection will become practically constant for the structure under consideration.

It is the practice to make girders slightly arched, or cambered, in order that when loaded they may not deflect below the horizontal line, which would give a very undersirable appearance, and the amount of camber allowed is 1 inch to every 40 feet of span, which in all ordinary cases will be found to be ample. Suppose a girder to be strained on top and bottom flange to 5 tons per square inch, and take $E = 7{,}000$ tons, and $d = \frac{l}{10}$; then $D = \frac{l^2(c+t)}{8\,d.\,E} = \frac{10.\,l(5+5)}{8 \times 7000} = \frac{l}{560}$, or about 1 inch to 47 feet. It is, however, very seldom that the deflection amounts to anything like this amount.

CHAPTER VII.

IRON ARCHES.

In the iron arch properly designed there exists only one kind of strain, compression, but the arch must be of such a form that the line of strain lies wholly within the depth of the arch, so in the first place the mode of determining the correct shape must be set forth. The thrust at the crown will be found from a formula demonstrated in a previous chapter, showing that the tangential strain on a circle or circular arc at any point is equal to its radius *at that point* multiplied by the radial pressure at the same point. At the crown of an arch the tangential thrust is evidently horizontal, and the load at that point acting vertically, acts at right angles to the tangent, and

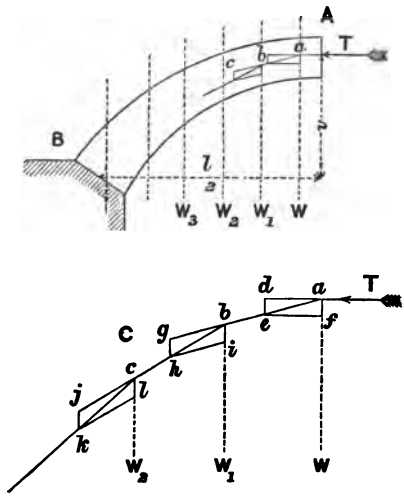

Fig. 44.

therefore radially. Let R = the radius of the arch in feet, and w = the load per (horizontal) lineal foot on the arch, then if T = the thrust at the crown, T = $w \times$ R. Let Fig. 44 represent half an arch, and suppose the length to be divided up into parts, each equal to $\frac{l}{20}$, where l = the span; let also v = the rise at the centre above the springings. The load on each of the longitudinal divisions will be $\frac{w\,l}{20}$. Let R = $l \times \cdot 6$, then the thrust T = $\cdot 6\, w.\, l.$ Through the centres of gravity of the loads $\frac{w\,l}{20}$, draw the vertical lines a W, b W, &c.

The direction of the horizontal thrust at the crown A is shown by the arrow T. Produce the arrow, and from a, where the produced line intersects a W, mark off to scale the horizontal thrust. This will be seen better on the enlarged sketch C. Make $a\,d$ = T, and $a\,f = \frac{w\,l}{20}$ or W; complete the parallelogram $a\,d\,e\,f$, then e will be a point in the curve of strain. Join $a\,e$, and produce it to intersect b W in C; make $b\,g = a\,e$, and $b\,i = \frac{w\,l}{20}$, or W_1; complete the parallelogram $b\,g\,h\,i$, then h will be another point in the curve; produce $b\,h$ to intersect c W_2 in c, make $c\,j = b\,h$, complete the parallelogram $c\,j\,k\,l$; k will be a third point in the curve, and similarly other points are to be found until the abutment B is reached, and through these points the line of thrust is to be drawn, and the arch so proportioned as to enclose it. If the load is not uniformly distributed, then W, W_1, W_2, &c., must be made to agree with the respective loads for each part.

In designing an arch, all the different loads coming upon it should be taken, and the lines of thrust worked out, and then the arch laid on so as to include them all.

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The loads are partly equalised on the arch by the members through which they pass in coming on to it. From a property of the right-angled triangle, viz. that the square of the hypothenuse is equal to the sum of the squares of the sides enclosing the right angle, an expression is found which will give the thrust at any point distant x from the crown of the arch. In the triangle $a\,d\,e$, $a\,d$ is horizontal, $d\,e$ is vertical, or at right angles to $a\,d$, and $a\,e$ is the thrust at a point midway between a and b, where the first length $\frac{l}{20}$ finishes. $a\,e^2 = a\,d^2 + d\,e^2$; but $a\,d = \mathrm{T}$, $d\,e = a\,f = \mathrm{W}$. Let $a\,e = t =$ thrust at point distant x from the crown, between which and the crown is the load W, then we have $t^2 = \mathrm{T}^2 + \mathrm{W}^2$, and $t = \sqrt{\mathrm{T}^2 + \mathrm{W}^2}$. Fig. 45 shows a diagram in which the thrust at the abutment is obtained without following the intermediate points. The load is uniformly distributed, then the centre of gravity of the half arch and load will be at $\frac{l}{4}$ from the crown, and making the thrust $\mathrm{T} = a\,b$, and $a\,d = \mathrm{W}$ (half arch), and completing the parallelogram $a\,b\,c\,d$, c will be the centre point of the abutment, and $a\,d = \mathrm{W} = v$. Let $\mathrm{W}^1 =$ the total weight on the arch, then $a\,d = b\,c = \frac{\mathrm{W}^1}{2} =$ the vertical component of the reaction of the abutment, and $\mathrm{T} = a\,b = \frac{l}{4}$; or by proportion, $\mathrm{T} : \frac{l}{4} :: \frac{\mathrm{W}^1}{2} : v$, $\therefore \mathrm{T} = \frac{\mathrm{W}^1 l}{8v}$, which is the same strain as would occur at the centre of the top flange

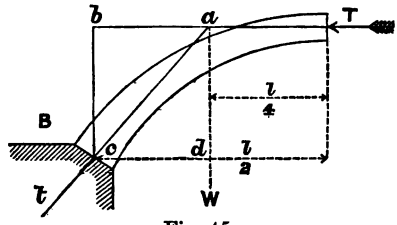

Fig. 45.

of a girder of span equal to the arch, and having a depth equal to the rise of the arch. From this the thrust at the abutment is found:—

$$t = \sqrt{\left(\frac{W^1 l}{8 v}\right)^2 + \left(\frac{W^1}{2}\right)^2} = \frac{W^1}{2}\sqrt{\left(\frac{l}{4 v}\right)^2 + 1}.$$

The stability of the abutment will be treated hereinafter, but when the abutments are held by a tie, the tension on that member will require to be determined, and this case will now be considered.

Fig. 46 represents a tied arch resting upon two supports, A and B. In this mode of construction the ends of the arch are fixed to metal abutments as shown, which abutments are prevented from spreading by the horizontal tie

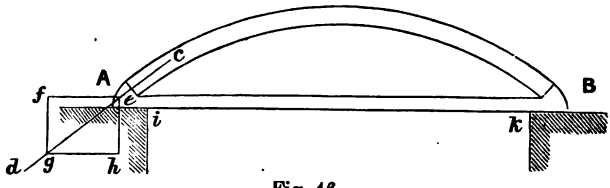

Fig. 46.

i k. Let *c d* be the direction of the resultant thrust on the abutment; this must be resolved vertically and horizontally —the latter for the strain on the tie. Let *e g* represent the thrust on the abutment, complete the parallelogram *e f g i*, then *f e* or *g h* will equal the horizontal pull on the tie.

But referring back to Fig. 45, it is seen that the horizontal component of the thrust on the abutment—that is, *a b* or *d c*—is the strain at the crown of the arch; hence the tension of the tie is equal to the thrust at the crown of the arch $= \frac{W^1 l}{8 v}$.

If the rise of the arch and its span be given, the corre-

sponding radius at the crown can be determined by equating the two formulæ for thrust at the crown thus: $\frac{W' l}{8 v} = w \times R$, but $W' = w l$; $\frac{w . l^2}{8 v} = w R \therefore R = \frac{l^2}{8 v}$. If the arc were circular the radius would be $= \frac{l^2}{8 v} + \frac{v}{2}$, according to the properties of the circle; the arc we have is a parabolic arc, of which the crown is the vertex.

The roadway may be carried either above or below the arch, the former being the most usual for arches with free abutments, the latter for tied arches. In either case the road girder, or the elements transmitting the load to the arch, should be sufficiently rigid to distribute concentrated loads, so that no undue distortion of the arch occurs. The amount of rigidity requisite will depend upon the ratio of the live or moving loads to the dead weight of the structure; the greater the latter is in proportion, the less will be the disturbing influence of the former.

We must now consider more particularly the effects of unequal loading upon the arch. The principle of moments may be applied to the calculation of the thrusts in this case, the same as it has been to the strains in those foregoing. I will first apply it to test the formula for the thrust at the crown.

Under a uniform load the centre of gravity of the half-arch will be $\frac{l}{4}$, from the nearest abutment, and the vertical reaction of the abutment (the vertical component of the thrust on the abutment) will be $\frac{w l}{2}$, acting at a horizontal distance $\frac{l}{2}$ from the crown. The vertical distance of the crown from the abutment is v; hence we have for the

moment of strain $\frac{w\,l}{2} \times \frac{l}{2} - \frac{w\,l}{2} \times \frac{l}{4} = -\frac{w\,l^2}{8}$; for the moment of resistance $T \times v$.

$T\,v = -\frac{w\,l^2}{8}$ $\therefore T = -\frac{w\,l^2}{8\,v}$, the same as above, with the negative sign showing the character of the strain; that is, compressive on the top member.

Let Fig. 47 represent the curve of thrust on an arch loaded only with its own weight and the dead weight of the roadway, and let this be 2 tons per lineal foot; the span 80 ft., and the rise 20 ft.; then the horizontal thrust at the crown will be $\frac{w\,l^2}{8\,v} = 80$ tons. Let a load $W =$ 5 tons come upon the arch at a point midway between the

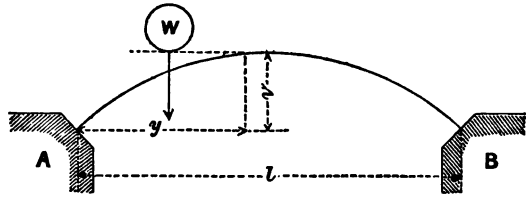

Fig. 47.

abutment A and the crown or centre. The virtual crown of the arch will not now be at the centre, and we must find its position, which will be at some point where the two parts of the structure exhibit equal positive moments about the abutments. The moments about A will be, if the point required be distant y feet from A, $\frac{w\,y^2}{2} + \frac{W\,l}{4}$, and the moment about B will be $w\,(l-y) \times \frac{l-y}{2} = \frac{w}{2}$ $(l^2 - 2\,l\,y + y^2)$; equating these, $\frac{w\,y^2}{2} + \frac{W\,l}{4} = \frac{w}{2}(l^2 - 2\,l\,y$ $+ y^2)$; $y^2 + 100 = 6400 - 160\,y + y^2$; $\therefore y = \frac{6300}{160} =$

39·375 ft.: this is where the *horizontal* components are equal.

Taking now the moments *about this point*, the horizontal thrust will be found. Let v' = the rise at this point; as it is so near the crown, in this case it may be taken as 20 ft. The reaction on B will be $\frac{w\,l}{2} + \frac{W}{4} = 80 + 1{\cdot}25 = 81{\cdot}25$. The weight of structure between B and the virtual crown is $w \times 40{\cdot}625 = 81{\cdot}25$, and this acts at a mean distance 20·3125 ft. Hence the thrust is, $\frac{81{\cdot}25 \times 20{\cdot}3125 - 81{\cdot}25 \times 40{\cdot}625}{20} =$

$\frac{1650{\cdot}39 - 3300{\cdot}78}{20} = -82{\cdot}52$ (nearly).

In the enlarged diagram, Fig. 48, is seen the amount of distortion at the point under W, and at a corresponding point in the other side of the arch due to the load W. *a* is the centre of the arch, and *b* the crown displaced; then *c c* is the curve of strain due to the uniform load, and *d d* is the distorted curve due to the presence of W. The curve is depressed under the load W, and elevated on the opposite side of the arch.

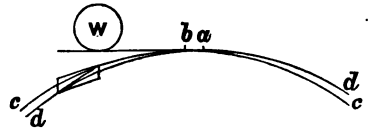

Fig. 48.

It seems advisable to insert the general equation of which we have worked out this example.

Let z = distance of W from abutment A, the other notation remaining as we have it above; then—

$$\frac{w\,y^2}{2} + W\,z = w\,(l-y) \times \frac{l-y}{2} = \frac{w}{2}(l^2 - 2\,l\,y + y^2);\ y^2 +$$

$$\frac{2\,W\,z}{w} = y^2 + l^2 - 2\,l\,y;\ \therefore y = \frac{l}{2} - \frac{W\,z}{w\,l}.$$

After the curves of strain due to the extreme loads have been determined, and the rib so proportioned as to include

them all, it only remains to adopt sectional areæ in accordance with the thrusts coming upon the different sections of the arch. Extending the above example, where it was found that under the uniform load the thrust at the crown was 80 tons, we find that the thrust at the abutment will be—

$$t = \sqrt{(80)^2 + (80)^2} = \sqrt{12800} = 113{\cdot}13 \text{ tons.}$$

Iron ribs are liable to expansion and contraction (the same as girders) under changes of temperature, and this occasions also change of form. In order to meet the requirements of the altering inclination of the ends of the arch, the faces of the abutments have been in many cases made curved; in some a convexly rounded end is supported in a concave abutment; in others, a slightly concave end rests on a slightly convex abutment, the radii being different; but this arrangement has the disadvantage of giving only a narrow band of bearing surface, and rendering the structure more liable to vibration than when a more ample bearing surface is provided. There is also some alteration of form, accompanied by a movement on the bearing surfaces, when, under the influence of a load, the arched rib shortens under the compressive strain, and the radial end surfaces approach more nearly the vertical position.

CHAPTER VIII.

SUSPENSION-BRIDGES.

In Fig. 49 is shown the general form of a suspension-bridge at A E. A B C D E is the main chain, to which the road girders are attached by the vertical suspending-rods; the main chain is securely fastened at A and E, and passes

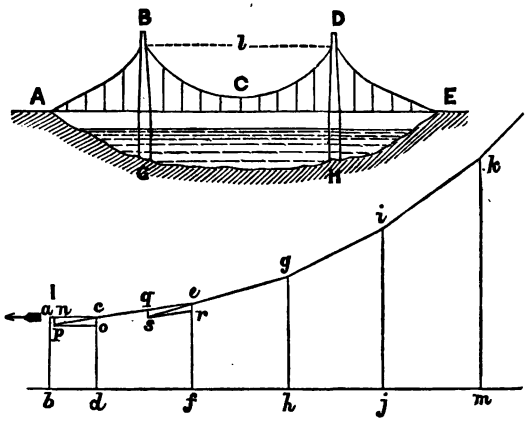

Fig. 49.

over saddles or rockers on the towers or piers B G and D H.

Let *a k* represent a part of the chain enlarged, *a* being the central or lowest point, and *b m* a portion of the road

girder, carried on the suspension-rods *c d*, *e f*, *g h*, &c. The strain at *a* may be calculated by a formula similar to that used for the crown of the arch, the only difference being that the chain is in tension and the arch in compression. If then l = the span in feet, v the fall of the chain at the centre, w the load per lineal foot, and T the horizontal tension at *a*, $T = \frac{w.l^2}{8\,v}$. On the horizontal line *c a* mark off *c n* = T, and on the vertical line *c d* mark off *c o* equal to the load on the rod *c d*; complete the parallelogram *n p o c*, then *p c* will be the strain resulting from T and *c o*, and its direction will be that proper for the link following *a c*; produce *p o* to *e*, the head of the next suspension-rod, and make *e q* = *c p*. Make *e r* = the load on the rod *e f*; complete the parallelogram *q s r e*; then *s e* will be the tension on and direction of the link *e g*, and in like manner the tensions and directions for the remaining links may be found. The horizontal strain must in the first place be found from the general load, in a manner precisely analogous to that employed in the case of the arch.

It is to be noticed that at the piers where the chain passes over a saddle or rocker the tension is equal on both sides of the pier, being unaltered by the change of direction, the same as when a strained rope passes over a pulley.

There being no rigidity in the chain, it will alter its form so as to suit its *centre line* to every variation of load; hence the suspension-bridge is intrinsically unstable. To render such a structure practically satisfactory, some means must be adopted to distribute the load as far as possible in a uniform manner; a stiff roadway girder at *b m* will effect this, and it is also facilitated by using two chains, one above the other, and attaching the head of the suspension-rod to a saddle-plate resting on the two chains, as shown at Fig. 50. *a* is the head of the suspension-rod, joined by a pin to the lower part of the triangular saddle-

plate, of which one of the upper corners is carried on the pin *b* of the lower chain, and the other is held by a pin *c* *resting* on the upper chain. The next suspension-rod will have its saddle carried by one of the ordinary pins of the upper chain, and by a pin *resting* on the lower chain. One saddle-plate must not be pinned to *both* chains, as that would interfere with their adapting themselves to varying loads, and so lead to vibration and distortion of the suspending-rods. As an arch, instead of being sustained by solid abutments, may have the thrust at the haunches opposed by a tie, so the pull of the chains, instead of passing away to anchorages at the ends, may be opposed by a strut or horizontal compression member passing from B to D (Fig. 49), and in that case the strain on such strut will be equal in intensity to the horizontal tension on the chain.

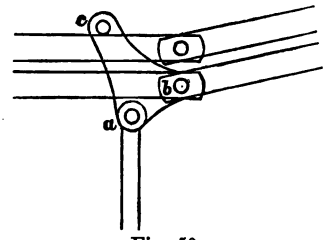

Fig. 50.

Also the arch and chain may be combined, as in the Albert Bridge at Saltash, the parts of the structure being so proportioned that the thrust of the arch is met and balanced by the pull of the chain.

The roadway may be above or below the chain, but is usually placed in the latter position.

Many varieties of suspension-bridges have been designed with the view of obtaining a structure more inherently stable than the ordinary form, and one of the most striking of these is what we may call the half-chain bridge, invented some years since, and possessing many features to recommend its adoption, although, probably from its being but little known, it does not appear to have come into use, at least in this country.

In Fig. 51, this arrangement of chain is shown in elevation at A, and in plan at B. The lower ends, *b* and *d*, of the semi-chains are secured to the lower parts of the towers over which the upper ends pass, being anchored in the ordinary way. In this system it is also to be noticed that the chains do not hang in vertical planes, the lower ends

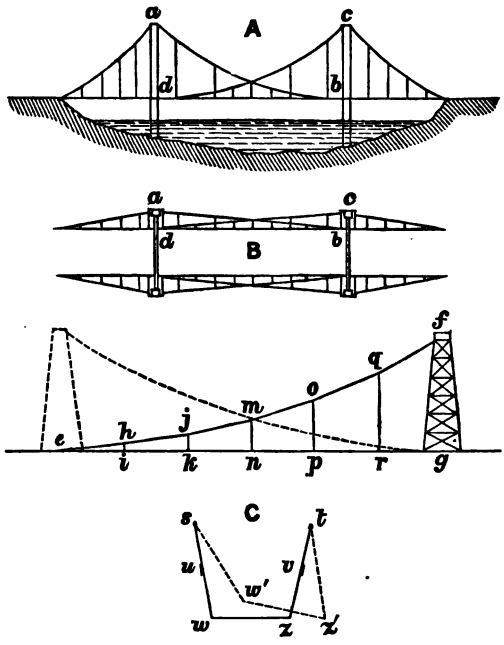

Fig. 51.

being brought closer to the centre of the roadway than the upper; and this secures more stability, as it throws the suspension-rods out of the vertical, as shown in the cross section at C, where *s* and *t* are the upper and *u* and *v* the lower chains, *s w* and *t z* being the suspension-rods. In

the ordinary suspension-bridge with vertical rods, if by any force the structure is caused to vibrate laterally, there is merely its own weight to resist such vibration, but in the section here shown any lateral movement causes the strain on one side rod to become greater than that on the other opposite it; hence a strong tendency to resume its normal position. The angular disposition of the rods will bring upon them strains greater than the load in the ratio of their lengths to their vertical heights, and this will lead to a lateral stress on the chains, tending to overturn the piers inwards, therefore they must be held in position by struts connecting the tops of the opposite towers.

e f represents our semi-chain, with its suspension-rods *h i*, *j k*, &c. The strains will be the same as on the half of a complete chain of the same form and fall, its span being double that of the semi-chain. In this bridge, as constructed, the road girder was not continuous, but consisted of a series of short longitudinal girders equal in length to the distance between two suspension-rods; and this to avoid the vibratory wave sent forwards in a continuous road girder by the rising of that girder in the bay next in front of that on which the load is entering; but the discontinuous arrangement has this disadvantage, that it does not distribute the load over several suspension-rods, as does the rigidly continuous road girder.

It is obvious that the semi-chain will be much less liable to continued pendulous vibration than the complete chain, and from its position, lying as it does in an inclined plane, it will have on it an initial strain, which is of great service under lateral disturbance; for if it be supposed that the platform is thrown aside until the suspension-rods occupy the positions shown by the dotted lines *s w'*, *t z'*, there is evidently a great effort on the part of the rod *s w'* to swing back with its load into the normal position, while at the commencement of such return there is no resistance offered

to it from the side s', although, as the platform approaches its proper place, the rising of the end s' of the rod $t\,s$ checks the onward vibration of the platform. In the ordinary bridge the suspension-rods appear to act like two isochronous pendulums connected by a link.

Before leaving this subject I must mention the anchorage of the main chains. The chain $c\,d$, Fig. 52, may be carried

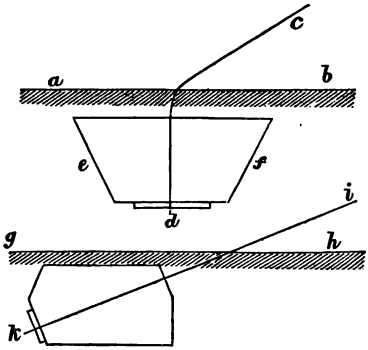

Fig. 52.

over a saddle, and so brought into a vertical direction, and passing below the ground line $a\,b$, be anchored beneath a mass of masonry $e\,f$, the weight of which should be at least twice the maximum strain that can be put upon the chain.

In another arrangement the chain is anchored behind a mass of masonry, as shown at $i\,k$, $g\,h$ being the ground line. In this case the stability of the masonry is relied upon; in both cases the work must be so executed that the masonry behaves as if it were one solid mass: the method of executing this part of the work will find its place in the chapter on Foundations.

CHAPTER IX.

COLUMNS AND STRUTS.

THE conditions under which materials yield and fail when subjected to compressive force are very various, and they have not been sufficiently ascertained to enable a rational theory of resistance to compression to be formed: hence empirical formulæ form the only resource. By empirical is meant a formula deduced directly from experiment, the laws of variation as well as the constants being obtained therefrom. As might be expected, the results so obtained are not so satisfying to the thoughtful mind as those derived from a combination of pure reasoning and experiment; but still in the absence of the latter we must, until more light shall be thrown upon the subject, be content to put up with the former.

Under certain known conditions the stress may be calculated, as for instance when a force acts rectilineally on a curved member, and passing outside its section, gives rise to a bending moment. It may be interesting to examine the effects of loads on straight elements, *assuming that they will bend* under the superimposed load.

Let there be a bar 3 feet 6 inches long and 1 inch square placed horizontally on supports 3 feet apart, and loaded at the centre with 0·2 ton; then, according to the formulæ in a former chapter, if the bar be of cast iron its deflection under this transverse strain will be,

$D = \frac{W\,l^3}{14.m} = \frac{\cdot 2 \times 3^3}{14 \times 1} = 0{\cdot}385$ inch; and the moment of strain of the load producing this deflection will be $M = \frac{W \cdot l}{4} = \frac{\cdot 2 \times 3}{4} = \cdot 15$ foot tons. The central moment of strain due to a force acting on the ends will be equal to the intensity of the force multiplied by the central deflection, or W′ D if W′ = the force on the end. To find then the end load to produce a central moment equal to the above, the two expressions must be equated, giving $M = \cdot 15 = W' D = W' \times \frac{0{\cdot}385}{12}$; $\therefore W' = 4{\cdot}67$ tons. This load, though acting compressively in direction, produces both tension and compression on the bar, and the tensile resistance will be the measure of strength of the bar, taking half the moment as being upon half the section; that upon the half in tension is 0·075 ft. ton, or 0·9 inch ton. The maximum tensile strain will be found by inverting the expression $M = \frac{1}{2 \cdot 6} s\,b\,d^2$; thus $s = \frac{12\,M}{b\,d^2} = 10{\cdot}8$ tons, which would exceed the strength of ordinary cast iron, unless the resistance of flexure is included (as explained in the chapter on Bending Stress), including which the tensile resistance, as found from the ultimate moment 8,000 inch lbs., becomes 48,000 lbs. per square inch, or 21·4 tons per square inch.

The deflection, however, will not be of the same character under compression by direct and transverse stress in respect to the moment of strain at any part; under the latter the strain commencing at each point of support increases simply as the distance from the point of support to its maximum at the centre; but under direct pressure the moment at any point is the load multiplied by the deflection at that point. Now it is easy to imagine a sample of iron having one place much weaker than the rest, or it

may be, from some defect in the casting, that it is not actually straight; then the strain may pass axially from each end to such a point, the deflection all occurring at that point. In saying the deflection all occurring at such a point, the additional deflection due to bending is intended, the form of the bar being somewhat of the form shown in Fig. 53; then wherever this point is situated will be the position of maximum deflection, and of maximum strain in consequence.

Fig. 53.

In the foregoing examination I have regarded the compressing strain as acting at the *centre* of the bar, but practically that will only occur while the bar *does not deflect*, for as soon as deflection commences the pressure will act only on the sides *a* and *b* of the end sections of the bar, thus reducing the moment. But in the first place the load must act centrally, for an initial pressure at *a* and *b* would deflect the bar in a direction the opposite of that shown.

The calculated (from experiment) resistance of such a column as that taken above is 8·1 tons, so if we consider the bar as breaking by deflection, it is clear that the tensile strength is not in this class of strain augmented by the resistance of flexure.

One very striking difference in the behaviour of the material under the transverse and the compressive deflecting forces must be noted. Under transverse strain the ultimate strength of the bar is not affected by variations in the modulus of elasticity, which only affects the amount of deflection; but under the compressive force the less the modulus of elasticity the greater the deflection, and therefore the greater the moment of strain under a given compressing force, and therefore practically the less the ultimate strength.

It is evident from this that the iron selected for elements

in compression should be of a quality exhibiting a high modulus of elasticity. When a bar does not fail by deflection and cross breaking, its rupture may occur by shearing at some plane more or less inclined to the axis, as at $g\,h$ or $f\,i$ in Fig. 54, representing part of a column of which $a\,b$ is the axis. Taking the action on the plane $g\,h$, make $c\,e$ = the load; this must be resolved in the directions of the line of fracture, and a line at right angles to it, the parallelogram being that shown at $c\,k\,e\,l$; and in like manner for the plane $f\,i$, the parallelogram will be $c\,n\,e\,m$, $c\,k$ and $c\,n$ being the shearing forces respectively acting parallel to the planes $g\,h$ and $f\,i$.

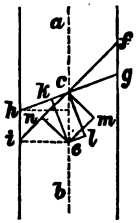

Fig. 54.

The maximum strain will be found to occur when the angle of the plane of rupture is 45 degrees to the axis of the bar, then the strain parallel to the plane of rupture will be $= \frac{W}{1{\cdot}414}$, and the area on which this strain acts is the sectional area of the bar at right angles to its length, multiplied by 1·414. It is evident, however, that this mode of rupture can only occur in columns of small length in ratio to the diameter, or least thickness, for if the resultant $c\,m$ fall *outside* the base of the column, there must be bending strain. Taking the plane at the angle given above, and resolving it at the centre or axis, the base of the column must have a width equal to its height.

From experiment it is found such a column would crush with 36 tons for a square inch. The proportion of this acting parallel to the plane of fracture would be $\frac{36}{1{\cdot}414} =$ 25·4 tons. The area of sheared section will be, for the bar 1 inch square, 1·414 square inches; hence the shearing

strain per square inch $= \frac{25{\cdot}4}{1{\cdot}414} = 18$ tons (nearly). This is a very high standard, but the experiments upon which the formulæ were based were made upon metal of superior quality.

A considerable difference in strength will be found to exist between elements having flat properly bedded ends and such as have jointed ends, or ends upon which the column can turn, for the flat ends aid in resisting deflection; and it may be observed, as in the deflection of columns the extended side is that on which the compressive load is most directly resting, there will be a great tendency for the deflection, when it does occur, to happen suddenly, perhaps instantaneously, with rupture, and so pass unnoticed.

I will now insert the empirical formulæ, which have been derived from the experiments of Hodgkinson and others. The formula for timber is Love's, those for metal are Gordon's. The diameter or thickness is always to be measured the thinnest way of the column; thus, a column 6 inches by 4 inches would be said to have a thickness of 4 inches for the purposes of calculation.

Let W = breaking load in tons per sectional square inch of column; r = the length divided by the least diameter; C = ultimate resistance to compression in tons per square inch.

For *Timber*, $W = \frac{C}{1{\cdot}1 + \frac{r^2}{418}}$.

For *Cast-iron cylinders*, solid or hollow, flat ends, $W = \frac{36}{1 + \frac{r^2}{400}}$; jointed ends, $W = \frac{36}{1 + \frac{r^2}{100}}$.

For *Cast-iron rectangular columns*, flat ends, $W = \frac{36}{1 + \frac{r^2}{500}}$;

jointed ends, $W = \frac{36}{1 + \frac{r^2}{125}}$.

For *Wrought-iron solid rectangular columns*, $W = \frac{16}{1 + \frac{r^2}{3000}}$.

For *Angle, Tee, and Channel Iron*, $W = \frac{19}{1 + \frac{r^2}{900}}$.

For *Mild Steel*, solid round pillars, $W = \frac{30}{1 + \frac{r^2}{1400}}$; rectangular pillars, $W = \frac{30}{1 + \frac{r^2}{2480}}$.

For *Strong Steel*, solid round pillars, $W = \frac{51}{1 + \frac{r^2}{900}}$;

rectangular pillars, $W = \frac{51}{1 + \frac{r^2}{1600}}$.

CHAPTER X.

JOINTS AND CONNECTIONS.

THE strength of any structure is limited by that of its weakest part, and in order to obtain the most satisfactory results all the parts should be equally strong. In practice it is not possible to secure absolute equality of strength throughout our work, but this should be studied as closely as is practicable, and at all events care must be taken that the strength nowhere falls *below a certain limit.*

The student, having made himself proficient in the foregoing formulæ, can readily determine the areæ of the various elements of any structure he may have intrusted to his care; but when this is done there arises the question of arrangement of joints for the connection of the parts, and the transmission of strain from one to another. The sizes in which materials can be obtained have to be considered, and the joints arranged so as not to interfere with one another. The lengths in which bars and plates are rolled vary in different districts; thus plates 21 feet long are common in the Cleveland District, whereas about 16 feet rules in Staffordshire. A great deal depends upon the quality of the iron and its peculiar characteristics, and it is not advisable to insist on excessively long plates, for by extending the dimensions the fibre of the metal may be strained in manufacture, or in avoiding this the maker may be led to use a class of iron of a more yielding character,

and inferior in strength throughout. Other points also require regarding, such as convenience of handling and erection; for girder work generally 20 feet should be taken as the outside limit of length for plates; but angle irons of moderate section may be run up to 30 or 35 feet in length, but this is rather awkward to manage, and it is more convenient to keep to shorter lengths.

For the sizes of timber no general rules can be laid down of any practical utility. I will commence with the joints of timber structures. The liability of timber to split along the line of the grain calls for great precautions in setting out the joints in this material. Joints in compression will be most satisfactorily made by butting the ends accurately together, as shown at *a b*, Fig. 55, and keeping them in juxtaposition by surrounding the joint with a box, shown in section at *e f*, and secured from slipping by bolts passing into or through the timber.

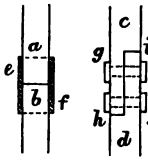

Fig. 55.

Another form of butt joint is shown at *c d*, in which the ends of the timber are stepped together and secured by bolts. This form is correct if the two pieces of timber are of exactly the same quality as regards elasticity; if not unequal straining may occur from the piece, say *g h*, being more compressible than the other tongue *j i*, when the one side yielding more than the other, the post will be more liable to deflect laterally. There is also more difficulty in insuring a fair bearing at *i* and *h* than there is in obtaining a uniform bearing in simple butt joints.

Joints subject to tensile stress will be generally more complex than those in compression, and in every case some sectional area will be lost. The most common method is by scarfing, as shown in Fig. 56. At *a b* is a plain scarf, bolted together by bolts passing through thin wrought-

iron plates *c c* and *d d*, of which the object is to spread the pressure of the bolt heads and nuts over the surface of the timber, and so prevent them from cutting into it, as they otherwise would. *e f* is an elevation of the side of the beam, having a plate on it. If these plates be not used, square wrought-iron washers should be placed under each head and nut, these washers being of a good size, such as 2½ inches square for a ¾-inch bolt, and so forth.

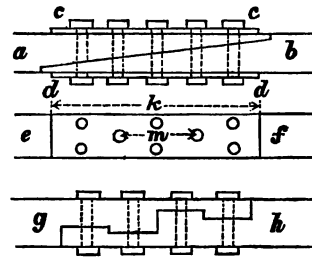

Fig. 56.

In this arrangement the whole of the strain is transmitted as shearing strain through the bolts. In the first place, therefore, the sectional area of the bolts must be proportioned to the tensile strength of the beam.

The strengths used in the examples will be taken from the working resistances given in the table at the end of the book.

Let the beam be of elm, 6 inches deep and 3 inches thick, then at 1 ton per inch its working strength will be $6 \times 3 \times 1 = 18$ tons. The shearing resistance of wrought-iron bars (from which the bolts are made) being 4½ tons per sectional square inch, the gross area of bolts required will be $\frac{18}{4 \cdot 5} = 4$ square inches. But we cannot get the full strength of the beam, for there will be a loss of section by the bolt holes; if there be two ¾-bolts opposite each other, as shown at *e f*, there will be 1½ inches taken off the width of the beam, and its working strength will be reduced to $4 \cdot 5 \times 3 \times 1 = 13 \cdot 5$ tons, and the area of bolts required will be $\frac{13 \cdot 5}{4 \cdot 5} = 3$ square inches. The sectional

area of a ¾-inch bolt is 0·44 inch, so the number of bolts of that diameter required will be seven, and for uniformity we should use eight, as shown in Fig. 56, and moreover it is advisable to have an excess of bolt area where it can be obtained without further reducing the area of the beam, as we cannot be certain that all the bolts will bear equally on the timber. So far, then, the strength of the bolts is secured, but failure may occur by their cutting out of the timber by detruding a piece equal to their diameter. If the work is sound, each piece of timber detruded will have to be sheared in two planes, through a distance k in the case of the outer row of bolts, and m in the central row. Should, however, the timber, by contracting on the unyielding iron of the bolts, crack, there will only be left one complete plane of shearing upon which to rely, and accordingly this one plane is all I shall take. The shearing resistance is ⅛th of a ton, hence the shearing area must be $= \frac{13{\cdot}5}{{\cdot}125} = 108$ square inches, and the area will be the breadth of the beam multiplied by the total length of the surface to be sheared. Let l = this length, then area $= 3 \times l = 108$; $l = \frac{108}{3} = 36$ inches. If the bolts in line are put 6 inches apart, centre to centre, and the timber overlaps 2 inches at the end, the shearing length will be for the two outer rows 28 inches, and for the centre 11 inches, making 39 inches in all—something in excess of that absolutely required.

In the mode of scarfing shown at $g\,h$, the detrusive resistance of the pieces hooked together is relied upon, and thus the cutting action of the bolts is avoided, the duty of the latter being to keep the surfaces of the joint in contact. It is evident that the sum of the lengths of the parts sustaining detruding force must (if the bolts are not to be regarded as assisting) be to the effective depth of the beam

as the resistance to tension is to the shearing resistance; that is, in the present case the line of detrusion must be 8 times the depth of the beam, therefore the length of the joint must be 16 times the effective depth of the beam. If, then, the beam loses 1¼ inches of its depth by the necessary cutting at the joint, the length of such a joint will be $16 \times 4{\cdot}5 = 72$ inches, or 6 feet: of course, if the strain is partly taken by the bolts, this length will be proportionately reduced.

In joints of beams under transverse strain, this will be partly compressive and partly tensile, and must be dealt with accordingly.

Connections of parts must now be examined. In Fig. 57, *a* is an upright connected with a horizontal tie *b*; it is tenoned in, and secured by a thin iron strap, which embraces the beam *b*, and is fastened to the upright *a* by bolts passing through it. Sometimes wrought-iron tie bolts are used, running the whole length of the upright, to tie the members together, but this does not seem to me as compact as the joint here described. *g* shows the end of a strut notched into the end of a tie *e f*, and held in position by a strap *h*, secured to the tie *e f* by a bolt *i*. Here the detrusive resistance of the timber at *e* is relied upon, and its area of detrusion must be proportioned to the strength of the tie *e f*, for it is evident that the horizontal strain put upon *e f* by the strut *g* will be that acting detrusively at *e*. In the lower sketch the end of the strut *k* abuts against a chock of wood or snug *n o*, bolted

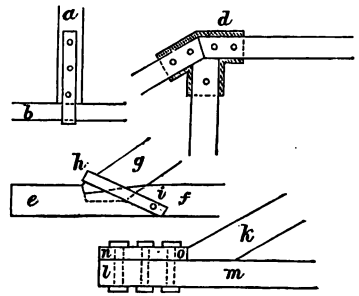

Fig. 57.

on to the tie beam *l m*; here the shearing resistance of the bolts is relied upon to carry the strain.

At *d* the ends of three timbers are shown as connected by being passed into a special casting made to receive them, where they are secured by bolts passing through the casting. The wrought-iron plates and straps to which I have referred are very thin, varying in thickness from $\frac{3}{16}$ inch upwards, according to the size of the timbers they connect. In such a place as *c* all the strap has to do is to support a portion of the weight of the tie beam *b*.

The strap *h* may be called upon to sustain the thrust of the strut *g*, in case the timber at *e* should give way and become detruded; hence in this position stronger metal will be required, and sufficient must be used to provide against such an accident, as the slipping of that strut would probably issue in the collapse of the whole structure of which it forms a part.

It is obvious that every care should be used to avoid as much as possible cutting into the timber, and so reducing its sectional area; and, moreover, if the sectional area be kept uniform, there will be less chance of the element becoming warped under the influence of damp or heat than if it be varied in dimensions.

I shall now pass on to the consideration of the joints and connections used in iron structures.

Let it be required to join two bars, *a* and *b*, under tensile

Fig. 58.

strain by a cover or joint plate *c*, Fig. 58. Let the bars be 4 inches wide by $\frac{3}{4}$ inch thick; the strain will tend to pull the bars apart by shearing through the rivets. Let the rivets be $\frac{7}{8}$ inch in diameter, then the area of each one

in cross section will be 0·6 square inch; the area of bar, less loss by rivet hole, multiplied by the working stress of 5 tons, or $(4 - 0{\cdot}875) \times 0{\cdot}75 \times 5 = 11{\cdot}71$ tons, which is the strain to be carried by one set of rivets from the bar *a* to the cover plate *c*, and again by another set of rivets from the plate *c* to the bar *b*. The working shearing resistance being 4·5 tons per square inch, the strength of each rivet will be $4{\cdot}5 \times 0{\cdot}6 = 2{\cdot}7$ tons; hence the number of rivets on each side of the joint will be $\frac{11{\cdot}71}{2{\cdot}7}$, or 5 rivets, for something more than 4 rivets being requisite, we cannot have less than 5, as shown in the figure. Had there been two joint plates, one on each side of the main bars, each rivet would have two sections acting; hence in this arrangement only half the number of rivets used with the single cover are necessary. It is necessary to consider the proportions of the various dimensions of the rivets, and to arrange them carefully, for upon the quality and disposition of this part of the work the safety of a structure depends, as much as upon the correct proportioning of the main elements. The rivets, when cold, must necessarily be of less diameter than the holes they are intended to fill, for they will of course expand when heated; hence care must be taken that when in the holes they are properly hammered or pressed up to fill them. Under ordinary circumstances, the rivets probably do not exactly fill the holes, and therefore hold the plates together against longi-

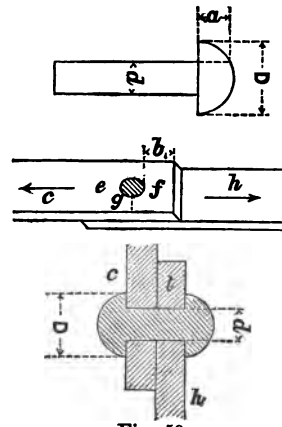

Fig. 59.

tudinal strain by friction, until by the vibration of strains the rivets are pulled so as to bear against the holes in which they rest. This will account for the leakage through rivet holes, and in fact leakage will show that the riveting is imperfect. In order to secure the filling of the rivet hole it is evident that the hammering up of the rivet should be continued until the metal of the plate surrounding the rivet body is of the same temperature as the rivet itself, so that it may *all shrink together* in cooling. Hand riveting seems more favourable to this result than power riveting, but the latter possesses advantages more than counterbalancing this.

I will first consider the necessary size of the rivet heads: the greatest strain to which the heads will be liable will be that due to the contraction of the bodies in cooling, and this will not exceed 10 tons per sectional square inch of rivet body, as that is the limit of elasticity, and at that strain the rivet will stretch. In the direction in which a rivet will pull out of its head, parallel to the fibres, the resistance does not probably exceed $\frac{2}{3}$ of the shearing strength across the grain; hence the working strain along the grain should be taken at $4{\cdot}5 \times 0{\cdot}8 = 3{\cdot}6$ tons per square inch. If d = diameter in inches of body of rivet, then the maximum pull on the head will be $0{\cdot}7854\ d^2 \times 10 = 7{\cdot}854\ d^2$ tons. The shearing area in the head will be the circumference of the body multiplied by the height a, Fig. 59; and its resistance at 3·6 tons $= 3{\cdot}1416\ d \times a \times 3{\cdot}6 = 11{\cdot}31\ d.\ a.$, as the resistance must be equal to the strain $11{\cdot}31\ d.\ a = 7{\cdot}854\ d^2$, $\therefore\ a = 0{\cdot}694\ d$, or the height at the place indicated by the dotted line should never be less than $\frac{7}{10}$ the diameter of the rivet.

This strain, moreover, must not put on the plates a greater compressing strain per square inch than the working stress of 3·5 tons, and to this the annular area of the under side of the rivet head must be adapted. Calling D

the diameter of the rivet head in inches, the annular area multiplied by compressive working stress will be $0{\cdot}7854\ (D^2 - d^2) \times 3{\cdot}5 = 2{\cdot}7489\ (D^2 - d^2)$, say, $2{\cdot}75\ (D^2 - d^2)$. Hence $7{\cdot}854\ d^2 = 2{\cdot}75\ D^2 - 2{\cdot}75\ d^2$, $\therefore\ D = 1{\cdot}96\ d$, which means, practically, that the diameter of the head should be twice that of the body of the rivet. A sufficient length must be allowed over that required by the thickness of the plates passed through to make the second head, and the least that can be put for this is $1{\cdot}4\ d$, or, practically, $1\frac{1}{2}$ diameters. If the quantity is slightly in excess, and forms a collar, this collar should not (as is sometimes done for appearance) be cut off, for in so doing the chisel will most likely cut into, and so weaken the plate upon which the rivet head bears, and the collar will not be noticed when the work is painted. If these proportions are adhered to, the rivet will be right for either shearing or tensile stress, which latter occurs when it hangs upon its head, and bears a longitudinal strain, which will not exceed 5 tons per inch, or one-half of that for which the head has been calculated.

Having determined the proportions of the rivet, the relations of the rivet holes to the bars or plates to be joined, and their distances from the edges and from each other, remain to be considered. The distance of the rivets in line from centre to centre is called the pitch of the rivets.

At *e* is shown a rivet in section in a hole near the end of the bar, the strain being in the direction of the arrows, tending to tear the end of the bar open. Rupture of the end of the bar may occur in different ways: the bar may tear open from the rivet to the end, or it may tear laterally through the dotted line *g*, or the rivet may push a piece out, though this latter is very improbable in iron (not being liable, like timber, to detrusion). The metal may also be damaged by the compression put upon it by the rivet. The part under compression at *f* will exhibit a section equal to the diameter of the rivet multiplied by the thickness of the

plate. Taking the stresses as before, shearing at 4·5 tons, and compression at 3·5 tons, and dealing with *one shearing section* only of the rivet, its strength will be $0{\cdot}7854\,d^2 \times 4{\cdot}5 = 3{\cdot}534\,d^2$. Let t = the thickness of the plate in inches, then its working strength to resist the compression from the rivet will be, $3{\cdot}1416\,d \times t \times 3{\cdot}5 = 11\,d\,t$; for these to be equal $3{\cdot}534\,d^2 = 11\,d\,t$, and the thickness of the plate must not be less than one-third of the diameter of the rivet.

Next, as to breaking through the line g; each side of the bar (if the rivet is central) will take one-half of the strain on the rivet, or $1{\cdot}767\,d^2$ tons. The resistance will be the length g multiplied by the thickness, and by 5 tons tensile strain $= 5.\,g.\,t = 1{\cdot}767\,d^2$, $\therefore\ g\,t = 0{\cdot}353\,d^2$. Hence if t were at its limit of $\frac{d}{3}$, $g \times \frac{d}{3} = 0{\cdot}353\,d^2$, $\therefore\ g = 1{\cdot}059\,d$. Finally, as to bursting out along the line f, it has been found that when the other proportions are properly adjusted in order to obtain equal strength on the line f, it must be equal to one and a half diameters, but in practice the thickness is usually more than $\frac{d}{3}$. The rupture along f is a kind of cross breaking, the load being the strain on the rivet, the reactions being the tension on the metal acting on each side of the rivet: if the breadth of the bar be three diameters, the conditions will be those of a beam loaded in the centre and *fixed* at each end, and having a span of two diameters; the moment of strain will be $\frac{3{\cdot}534\,d^2 \times 2\,d}{8}$ $= 0{\cdot}883\,d^3$; that of resistance $= \frac{4{\cdot}\,5.\,t.f^2}{6} = 0{\cdot}75\,tf^2$. Let $t = \frac{d}{3}$, its least thickness, then $0{\cdot}883\,d^3 = 0{\cdot}75\,tf^2 = 0{\cdot}75\frac{d}{3}f^2$; $0{\cdot}883\,d^2 = 0{\cdot}25f^2$; $f = \sqrt{3{\cdot}53\,d^2} = 1{\cdot}88.\,d.$

These observations on the relation of rivet diameter to position of rivet holes in plate will also apply to bolt holes and pin holes.

In Fig. 60, A B shows in plan two main plates united by rivets disposed in a zigzag form, the object being to have several rows of rivets without much loss of sectional area in the main plates, for every rivet hole represents the area of so much waste metal running the whole length of the main plate, when the joints are in tension; in compression, of course, there is no loss by rivet holes, provided the rivets fill them. When the rivets are zigzag it must be arranged

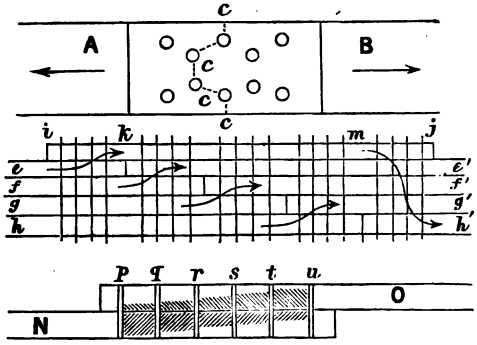

Fig. 60.

that no area that might tear, such as *c c c c*, for instance, is less than the effective transverse section of the plate; that is to say, a line *c c c c* passing through the centres of the rivets must not be less than the breadth of the plate, plus two rivet diameters in this case, for on the breadth two rivet diameters is the loss, and on the zigzag line four rivet diameters. When there is no zigzagging, but the rivets are in even rows, from four to six diameters are found to give a satisfactory pitch in practice.

In the second sketch *e e'*, *f f'*, *g g'*, *h h'* are four main plates,

having their joints all brought together under one cover; in this arrangement, the distance between any two joints must be sufficient to allow of the insertion of the number of rivets necessary for one plate (the plates here are all assumed of the same size). The way in which the stress passes is thus: that from h passes to g', from g to f', and so on as shown by the arrows; that from e passing into the cover plate ij through the rivets between i and k, and out again through the rivets between m and j into the plate h

At N O is shown the way in which the strain may be supposed to pass from one plate to another, being taken up step by step by each line of rivets as they are approached, and so through the length of the plates, increasing on one as it diminishes on the other, as illustrated by the shaded parts, the shading indicating the amount of stress on the plates at each point.

In butt joints in compression, strictly speaking, no stress should come on the rivets, but if the ends of the plates do not absolutely butt against each other, the whole stress passes through the rivets; hence it is advisable (as butt joints are not easy to get) to provide a sufficient number to carry it.

The punching of the plate is found to weaken it to a varying extent; that is to say, it reduces the strength of the metal left, probably because in this process the piece removed is burst out rather than cut, and for some distance round the hole the material will be strained permanently, and by its distortion may also put a permanent strain upon the metal beyond the zone actually injured by the action of the punch. It has been found that annealing the metal restores its strength, and this indicates that the deterioration is due to local molecular disturbance. In my opinion the best plan is to punch the holes smaller than they are required, and drill out the injured zone, and about $\frac{1}{8}$ inch will be found sufficient for ordinary thicknesses. The sharp

edges left round the hole by the drill should be rounded off, otherwise the shearing resistance of the rivet will be reduced, as the edges of the holes will act as veritable shears on account of their sharpness, and from this cause alone there may be as much as 5 per cent. loss of strength on rivet area.

I will now examine the proportions required for screw bolts; as these are put in cold they are not liable to the exceptional strains coming upon rivets. Let as before d = diameter of body in inches, D = diameter of head (that is, the least width over the head), H = height of head, h = height of nut, all in inches; working strains as before.

The ratio of shearing area to resist drawing of the body of the bolt out of the head will, as there is no greater strain on it than the ordinary working stress, be to the cross section of the bolt as 5 is to 3·6; hence $3{\cdot}6 \times 3{\cdot}1416\, d \times H = 5 \times 0{\cdot}7854\, d^2$, whence $H = 0{\cdot}348\, d$. Common practice does not make them less than half the diameter in height, and h, the height of the nut, is usually equal to the diameter, this margin being allowed for loss by cutting the thread, and the fibre being cut transversely through, is materially weakened in the process. The bearing area of the head of the bolt or the nut will be to the cross section as 5 to 3·5. $5 \times 0{\cdot}7854\, d^2 = 3{\cdot}5 \times 0{\cdot}7854\, (D^2 - d^2)$; $D = 1{\cdot}56\, d$.

Bolts subject to strain must have their threads carefully cut in a lathe by a proper chasing tool, the threads of the nuts similarly made, and to fit the bolt threads, and under no circumstances must *die-cut threads* be permitted where there is any strain, for these threads are partly cut and partly squeezed up, as may at times be noticed by the slight furrow on the top of the thread, showing that the sides have burred up under the pressure of the dies, and the most ordinary common sense will indicate that such threads must be quite useless under strain.

Fig. 61 shows a joint made by a gib and cotter, *c* and *e*. Two bars, *a a'*, are jointed to the bar *b*. In each bar is a rectangular hole, into which the key or gib *c* is inserted, being then tightened up by driving in the cotter *e*. The slight taper on the back of the gib and one side of the cotter allows them to be driven tight, while the surfaces pressing against the inside of the holes in the bars remain parallel. As there is some liability to work loose where much vibration occurs, the cotter is sometimes prolonged, as in *f*, into a screw, which, passing through a ring carried on a prolongation *g* on the gib, is secured by a bolt *h*, but this precaution is usually confined to machinery. The gib and cotter arrangement possesses advantages of tightening up joints that recommend it in the eyes of many for the joints of counterbracing.

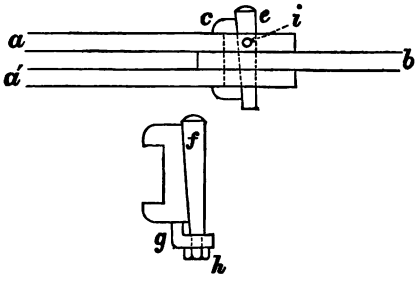

Fig. 61.

The cotter, when driven up, may also be kept from moving by drilling a small hole through the work, and driving in a pin, and similarly a nut, when screwed up, may be kept from turning; but this does not permit any future tightening up of either connection.

In Fig. 62 is shown an ordinary joint of a cross girder with a main girder. A is the bottom flange of the main girder, and B is the cross girder; it rests upon A, being riveted at *a a a* to one of the upright stiffeners, which is turned at an angle so as to fit on the top of the cross girder; it is connected with the web by the rivets *b b*, and steadied on the main girder flange by *c*. This is as good a joint

as can be got for the purpose, and affords also a good steadying resistance to the lateral forces acting on the main girder. The rivets *a a a* are in tension; *b* and *b*, under shearing strain; *e* also is in tension if any part of the load comes directly on the flange A. If the cross girder is attached under the flange, the rivet heads alone are relied upon to support it—a thing to be avoided where possible.

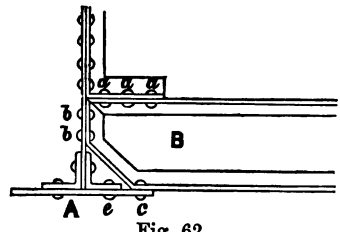

Fig. 62.

I will now remark upon the connections of the various parts of the main girders themselves, and point out the nature of the strains occurring there.

The relations between and equilibrium of the flanges are maintained through the web, whether the girder be of plate or lattice web, and the connections of these three parts—the two flanges and the web—are made by the rivets passing through the angle irons. Taking any point in either flange, we find the flange joined to its angle irons by two rivets, in a line transverse to the length of the flange; therefore there are two rivet areæ in shear, and corresponding to these the two angle irons have *one rivet* joining them to the web; but as the web is between the vertical limbs of the angle irons, this rivet must be sheared in *two places* before the web can break away; hence here there are also two rivet areæ in shear, so that the strength is uniform throughout the connection. The strain on a girder increases from the point of support towards that of maximum strain, and it is evident that each pair of angle-iron rivets must take up the increase of strain accruing from its increased distance from the point of support. Let there be a girder 100 feet span carrying 1·5 tons per lineal

foot, and being 8 feet deep, and let the rivets have a pitch of 4 inches; what, then, will be the strain on the first pair of rivets? When $x = 4$ inches, $\frac{w x^2}{2 d} - \frac{w l x}{2 d} = \frac{1{\cdot}5 \times (\frac{1}{3})^2}{2 \times 8} - \frac{1{\cdot}5 \times 100 \times \frac{1}{3}}{2 \times 8} = -3{\cdot}02$ tons. This is the place of the most rapid increase of strain, and therefore of the maximum strain on the rivets; the above would require $\frac{3{\cdot}02}{4{\cdot}50} = 0{\cdot}66$ square inches of rivet area; two $\frac{3}{4}$-inch rivets would give 0·88 square inches of area, hence would be ample for the purpose. Practically this size and pitch will be kept throughout the girder.

Next, we must consider the duty of the rivets holding the flange plates together when there are more plates than one. As the plates farthest from the neutral axis are the most strained, it is evident that by their elastic resistance they will tend to slide upon those next beneath them, and in so doing will bring shearing strain upon the rivets holding the plates together, this strain being equal to the difference of strain on two contiguous plates. To take an extreme case, let the girder be 12 inches deep over all, and each flange to consist of two plates 12 inches wide and $\frac{3}{4}$ inch thick each, then the centres of these plates will respectively be 5·625 inches and 4·875 inches from the neutral axis; so, if the outer plate have a strain of 5 tons per square inch upon it, its total strain will be, deducting the loss by rivet holes, $(12 - 1{\cdot}75) \times 0{\cdot}75 \times 5 = 38{\cdot}43$ tons, and that of the plate next $38{\cdot}43 \times \frac{4{\cdot}875}{5{\cdot}625} = 33{\cdot}3$ tons, the difference being 5·13 tons, to be carried by each pair of rivets, and therefore requiring $\frac{5{\cdot}13}{4{\cdot}5} = 1{\cdot}14$ sectional square inches of rivet area. The two $\frac{7}{8}$-inch rivets for which allowance has been made in taking the strength of

the plate will give 1·2 inches area. To the above strain is to be added the increment due to the increasing strain on the flange; but this near the point of maximum strain is very trifling, and as it increases towards the ends the strain just taken will generally fall. It may be advisable to see what is the actual increment of strain at 4 inches pitch. If the girder be 20 feet span, the load to give the strain mentioned on the plates must be 25 tons, or 1·25 tons per lineal foot; the two values of x, to give the increment of strain at the centre, will be 9 feet 8 inches and 10 feet, which, using the same formula as employed in the previous case, gives for the two plates 71·43 tons, and 71·73 tons, the difference being 0·3 ton, which has again to be divided between the two plates, as it is only the portion passing to the outer plate that affects the section of rivet with which we are dealing, and that portion would be 0·15 ton, which would raise the required sectional area to 1·18 square inches.

In all this the resistance of the plates to sliding due to their friction has been (as is usual in practice) neglected, as we cannot be sure of its value, although it has been shown by experiment to exist, and to be more than equal to the strength of the rivets. This was proved by riveting up some plates, and having the rivets in large oval holes, when it was found that the rivets failed before the plates slipped for them to take their bearings.

The amount of frictional resistance between the plates of bridge structures may diminish with time, by the slackening of the initial tension of the rivets, for if they are assumed, when closed up, to have a tension of 10 tons per sectional inch, yet not only will they yield further under the vibrations, but in time the molecules themselves will, to some extent, rearrange themselves, and so reduce the stress to which the friction is due. If there be a tension of 5 tons per sectional inch of rivet area, the corresponding

frictional resistance of the plates, supposing them to be well flattened so as to bear evenly upon each other, would be 1·5 tons per inch of rivet area.

In Fig. 63 are shown examples of joints used in different parts of iron roofs. A is the connection of a tee iron strut with a rafter by means of two joint plates laid one on each side of the web of the strut, and having the vertical limb of the rafter between them. B shows another joint of this sort, but with the addition of the upper forked end of a suspension-rod, *e*, which embraces the joint plates, and one bolt fastens all together. *f h g* is the joint at the top of the roof where the two principal rafters meet. Two plates are put one on each side of the vertical limbs of the rafters, and riveted through as shown, and also carry between them the eye of a suspension-rod, *i*, where such a rod occurs in the construction, this rod hanging on a bolt as shown.

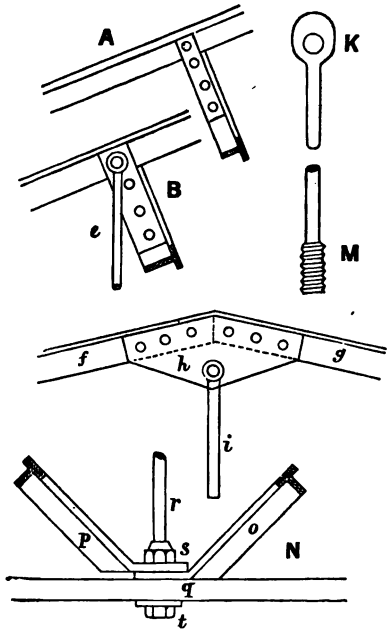

Fig. 63.

N is one of the lower joints, being that at the centre; *r* is the central suspension or king rod; *o* and *p* are two tee iron struts, of which the ends of the tables are turned up to a horizontal position, and drilled to admit the screwed

end of the rod *r*, which also passes through a hole in the main tie *q* of the roof, all these elements being fastened and held together by the nuts *s* and *t*.

K illustrates the general form of an eye at the end of a rod, and M the screw at the other end, which is to be cut on a part of greater diameter than the rest of the bar, so that the diameter at the base of the thread shall not be less than that of the body of the rod.

Between a nut and the surface upon which it presses should be interposed a metal ring or washer on which the nut will bear, and which will prevent the nut from cutting into the element beneath when it is being tightened up.

It may here be observed that where punching without subsequent drilling out of the holes is the mode of manufacture adopted, spiral punches should be used, as it is found that from their comparatively gradual action the general injury done to the metal is comparatively trivial: they have far more of a cutting action than the ordinary flat punches, and indeed it is as unreasonable to make the cutting edge of a punch horizontal as it is that of a shear, and no practical man would think of bringing the whole length of a shearing edge into action at once. The violent commotion caused amongst the particles of the iron by the flat punch is evident from its texture in the neighbourhood of the punched hole being frequently changed from fibrous to semi-crystalline.

In the connections made with cast iron by means of bolts passing through lugs very great care is called for in the execution of the castings, in order that the lugs may be perfectly sound and not cracked in the re-entering angle, which is apt to occur if the moulds are opened too soon. In all elements of this description the holes should be very carefully drilled to template (or pattern), for cast iron is too rigid to be wrenched into place as is occasionally, though improperly, done with wrought iron; hence, if the

bolt holes do not fit, one of them will be enlarged to let the bolt in, with the result generally of rendering that bolt useless.

In cast-iron piers braced with bars pockets may be formed to receive the ends of the bars, but in these cases the pockets should be made without a bottom; there should be a clear passage through for the escape of any water that might otherwise accumulate, for if not, the water may accumulate and freeze, and by its expansion in so doing cause the rupture of the pocket; this has been known to occur, and is of course attended by great trouble and expense.

The student must be cautioned against putting in joint plates where they are not required, for in such localities harm may result from their presence. For instance, if there be a number of single spans succeeding each other, each span being designed as a single girder; if then by a joint plate it is connected with those at each end, it becomes more or less a girder fixed at the ends instead of freely supported there, and thus the distribution of the stresses throughout the structure will be altered, and therefore will not be such as have been provided for in the design.

It is, as far as manufacture is concerned, very important for the supervising engineer to see that all the connections made during the erection of a structure are properly disposed and adjusted, for it sometimes happens that by ignorance or want of thought on the part of a workman a properly designed work is very unduly strained, either from mode of lifting during erection, or from the condition in which it is left when that process is complete.

CHAPTER XI.

COMBINATIONS OF GIRDERS.

ALL structures of magnitude carried by girders will consist of an aggregation of these elementary parts, but the term "combination" is here applied to indicate such an arrangement of the girders that they assist each other, or by their connection relieve or modify the strains upon each other. An ordinary bridge consisting of main and cross girders will be a simple aggregation of girders; the cross girders carry the roadway, and are in turn carried bodily by the main girders; the latter form the supports for the former, but in no way reduce or modify the strains upon them.

If a load is equally distributed over a number of girders identical in construction, they will all deflect by the same amount, and under other distribution the deflections will be as the loads. Now although it has been shown that the modulus of elasticity for wrought iron is a very variable quantity, yet it may be assumed that if all the girders in a given structure are made of the same kind of iron, and are under the same strain per sectional square inch, that in this structure the modulus of elasticity may be considered constant for all its parts, and the relative deflections will then follow the general laws, which we have found in the chapter on Deflection to be as follows:—

The deflection varies directly as the load, and as the cube of the span of the girder or length of the cantilever,

and inversely, as m, where $m = b\,d^3 - b'\,d'^3 - \&c. - b^n\,d^{n3}$. If a girder carrying a uniformly distributed load is partly supported at the centre, the deflection at the centre will be the total deflection due to the uniformly distributed load, less the deflection (upwards) due to the sustaining force at the centre. This has been proved by experiment. If, for instance, there is a girder 30 feet span loaded with 20 tons, the value of m being 44,000, the deflection under this load will be $D = \frac{W\,l^3}{44{\cdot}8\,m} = \frac{20 \times 30^3}{44{\cdot}8 \times 44{,}000} = 0{\cdot}274$ inches.

If there is a supporting force in the centre of the beam equal to 6 tons, the deflection equivalent to this will be $D = \frac{W\,l^3}{28\,m} = \frac{20 \times 30^3}{28 \times 44{,}000} = 0{\cdot}131$ inch; hence the actual central deflection of the beam will be $0{\cdot}274 - 0{\cdot}131 = 0{\cdot}143$ inch. The point of maximum strain will not be at the centre, but there will be two points of maximum strain, one on each side of the centre of the span, the curve of strain being as it were caught up at the centre.

Let l = span of the girder, w = load per lineal foot, R = reaction at one end point of support, P = upward sustaining force at the centre of the girder, M = moment of strain at any point distant x from the nearest end support. On each end support one-half the total load, less one-half the central force P, will act; therefore $R = \frac{w\,l}{2} - \frac{P}{2}$; $M = \frac{w\,x^2}{2} - R\,x = \frac{w\,x^2}{2} - \frac{w\,l\,x}{2} + \frac{P\,x}{2}$. The points of maximum moment of strain must now be determined. When the moment of maximum strain is reached, and that moment is about to be diminished, we may imagine an indefinitely small increase of x during which the moment remains constant; here, then, the increase of the positive quantity must equal that of the negative quantity. Let, then, x become $x + a$,

a being indefinitely small: $M = \frac{w}{2}(x+a)^2 + \frac{P}{2}(x+a) - \frac{w\,l}{2}(x+a) = \frac{w}{2}(x^2 + 2\,a\,x + a^2) + \frac{P}{2}(x+a) - \frac{w\,l}{2}(x+a)$.

The quantity a being originally very small compared to x, a^2 is so much smaller that it may be neglected; then the increase of the positive quantity is $w\,a\,x + \frac{P\,a}{2}$, and of the negative $\frac{w\,l\,a}{2}$; equating these, $w\,a\,x + \frac{P\,a}{2} = \frac{w\,l\,a}{2}$, $\therefore x = \frac{w\,l\,a}{2\,w\,a} - \frac{P\,a}{2\,w\,a} = \frac{l}{2} - \frac{P}{2\,w}$, which is the value of x, corresponding to the maximum moment of strain.

Suppose now a uniformly loaded girder to be assisted by another girder of similar section and span placed immediately beneath it, but supporting it only at the centre, the deflections of the two girders would be equal; what would be the relations of the maximum strains upon them? The maximum strain on the second girder (loaded at the centre) will be at the centre, and its moment there will be $M' = -\frac{P\,l}{4}$. The deflection on this girder is $D = \frac{P\,l^3}{28\,m}$. Let $w\,l = W$, then deflection at centre of first girder will be $D = \frac{w\,l^4}{44{\cdot}8.\,m} - \frac{P\,l^3}{28.\,m}$. But as these two deflections are equal, $\frac{w\,l^4}{44{\cdot}8.\,m} - \frac{P\,l^3}{28.\,m} = \frac{P\,l^3}{28.\,m}$, and $\frac{w\,l^4}{44{\cdot}8.\,m} = \frac{P\,l^3}{14.\,m}$, whence $P = 0{\cdot}3125\ w.\ l.$ The maximum strain on the first or uniformly loaded girder will be $M = \frac{w}{2}\left(\frac{l}{2} - \frac{P}{2\,w}\right)^2 - \frac{w\,l}{2} \times \left(\frac{l}{2} - \frac{P}{2\,w}\right) + \frac{P}{2}\left(\frac{l}{2} - \frac{P}{2\,w}\right) = \frac{1}{4}\left(P\,l - \frac{w\,l^2}{2} - \frac{P^2}{2\,w}\right)$; that on the second $M' = -\frac{P\,l}{4}$; and if r = the ratio between these strains, $r = \frac{M}{M'} = 1 - \frac{w\,l}{2\,P} - \frac{P}{2\,w\,l}$. If we desire to

divide the load in any particular manner, there are two ways of doing it—by interposing some element between the girders, so that their deflections are not equal, and by making the girders, of different sections, so that m has not the same value for both.

Let it be required to have the maximum strain per sectional square inch the same for both girders.

First let some intermediate element be used, so that the deflections of the two girders are not necessarily equal; then retaining the sections alike, the maximum moments must be equal. $\frac{P\,l}{4}-\frac{W\,l}{8}-\frac{P^2\,l}{8\,W}=\frac{P.\,l}{4}$; whence $P=0{\cdot}268.\,W.$ The interposed element must under the load P have a deflection equal to the difference of deflection of the two principal girders.

In the second case, for the sake of simplicity, let solid rectangular beams be assumed to be used, and the deflections equal. It is evident that the force P must produce half the deflection, as a central load, that the load W would produce uniformly distributed. Let D' = the actual deflection of the two girders, then $D'=\frac{1.}{2}\frac{W\,l^3}{44{\cdot}8.\,m'}$; also $D'=\frac{P l^3}{28.m}$; whence $\frac{W\,l^3}{89{\cdot}6.m'}=\frac{P\,l^3}{28\,m}$, where m' refers to the uniformly loaded girder, and m to the auxiliary centrally loaded girder: $28.\,W.\,m=89{\cdot}6.\,P.\,m'$. Some ratio between W and P must be decided upon, or else between m and m'; let $P=\frac{W}{2}$, then $28.\,W.\,m=89{\cdot}6.\,\frac{W}{2}.\,m'$; $\therefore\ m=\frac{89{\cdot}6\,m'}{56}=1{\cdot}6\,m'$. From this we have $b\,d^3=1{\cdot}6.\,b'.\,d'^3$; but the same strain per sectional square inch is also required. The maximum moment of strain on the first girder is $M'=\frac{P\,l}{4}-\frac{W\,l}{8}-\frac{P^2\,l}{8\,w}=\frac{W\,l}{8}-\frac{W\,l}{8}-\frac{W\,l}{32}=-\frac{W\,l}{32}$; the maximum

strain on the second or auxiliary girder $M = -\frac{P l}{4} = -\frac{W l.}{8}$ The moment of resistance, therefore, of the second girder must be four times that of the first, or $\frac{s b d^2}{6} = \frac{4 s b' d'^2}{6}$, and s, the strain per sectional square inch, must be the same for both; hence $\frac{b d^2}{6} = \frac{2 b' d'^2}{3}$; $\therefore b d^2 = 4 b' d'^2$. It has been shown above that to satisfy the conditions $b d^3 = 1{\cdot}6.\ b' d'^3$: hence, as equals divided by equals must be equal, $\frac{b d^3}{b d^2} = \frac{1{\cdot}6.\ b' d'^3}{4.\ b' d'^2}$, $\therefore d = 0{\cdot}4\ d'$.

For example, let the span of the beam be 6 feet, and the distributed load 1·2 tons, then $P = 0{\cdot}6$ tons; the moment of strain on the first girder $= -\frac{W l}{32} = -\frac{1{\cdot}2 \times 6 \times 12}{32} =$ 2·7 inch tons; on the second girder $M = -\frac{W l}{8} = -\frac{1{\cdot}2 \times 6 \times 12}{8} = 10{\cdot}8$ inch tons.

The moments of resistance must be equal to these moments of strain; hence if $s = 4$ tons, and $b' = 1$ inch, $\frac{s b' d'^2}{6} = \frac{4 b' d'^2}{6} = \frac{2 d'^2}{3} = 2{\cdot}7$, $\therefore d'^2 = \frac{7{\cdot}1}{2} = 3{\cdot}55$ inches, and $d' = 1{\cdot}884$ inches; $d = 0{\cdot}4\ d' = 0{\cdot}7536$ inch; $b d^2 = 4 b' d'^2$; $\therefore b = \frac{4.\ b' d'^2}{d^2} = \frac{4 \times 1 \times 3{\cdot}55}{0{\cdot}568} = 25$ inches. The deflections, then, of these two beams should be equal, if our deductions are correct: in the case of the first beam $m' =$ 6·688, and $D' = \frac{W l^3}{89{\cdot}6.\ m'} = \frac{1{\cdot}2 \times 6^3}{89{\cdot}6 \times 6{\cdot}688} = 0{\cdot}43$ inch. For the second or auxiliary beam $m = 10{\cdot}697$, and $D' = \frac{W l^3}{56.\ m} =$

$\frac{1{\cdot}2 \times 6^3}{56 \times 10{\cdot}697} = 0{\cdot}43$ inch. This, then, proves the accuracy of the foregoing investigation. It will be noticed that the ratio of the depths is apparently independent of the breadths, but these are ruled by the moment of strain to be sustained.

One of the commonest applications of this principle is found in the distributing girders of railway bridges, where, under a maximum load of engines, every alternate girder only will carry a direct load, and the distributing girder is therefore used to carry a part of the strain on to those girders that would otherwise be idle. This distributing girder is usually carried along the centre of the bridge, being firmly secured to every cross girder.

Now as each cross girder will, under a passing load, be alternately loaded directly, and acting as an auxiliary, it follows they must all be of the same section; the deflections therefore will be different, and the difference must be equivalent to the deflection of the distributing girder. If this be worked out according to the principles set forth above, and l be the span of the cross girders, and s the distance between them from centre to centre, and m apply to the cross girders, and m' to the distributing girder, it will be found that $m' = 6.\ m\left(\frac{s}{l}\right)^3$. This ratio cannot well be obtained in practice, as will be now shown by an example.

We have found that if the girders are of the same section, and have the same maximum strains upon them, $P = 0{\cdot}268\ W$. Let the bridge carry two lines of railway, and the load on each wheel of a locomotive be 7·5 tons (I am here assuming a train of tank engines, weighing 45 tons each, and carried on three pairs of wheels coupled and equally loaded), then on the directly loaded girder there will be 30 tons, and this is so distributed that we may practically consider it as uniformly distributed load.

The dead load is equally divided amongst the cross

girders, so there is only that part of the section required to carry the running load to be dealt with here.

Let the cross girders be 25 feet span, and 18 inches deep between the centres of gravity of the flanges. The load carried by the distributing girder on to the intermediate girder will be $30 \times 0{\cdot}268 = 8{\cdot}04$ tons. This will really go from the loaded cross girder, half in each direction, to the idle girders on either side; but as each idle girder will receive a like load from the loaded girders on either side, for the purposes of calculation, all the load taken from one girder may be regarded as placed upon the centre of the next.

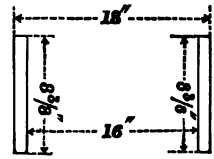

Fig. 64.

The strain on either flange will be, from this central load, $\frac{W.l}{4.d} = \frac{8{\cdot}04 \times 25}{4 \times 1{\cdot}5} = 33{\cdot}5$ tons: taking 4 tons per square inch as working strain, the effective area required will be $\frac{33{\cdot}5}{4} = 8{\cdot}375$ square inches, and the section required by the running load will be as shown in Fig. 64. The value of m will be, $m = b\,d^3 - b'\,d'^3 = 8{\cdot}375 \times \overline{18}^3 - 8{\cdot}375 \times \overline{16}^3 = 14539$. In the case I have taken, the wheel centres will be about 8 feet, so the cross girders will be 4 feet apart, or $s = 4$ feet; then $m' = 6\,m\left(\frac{s}{l}\right)^3 = 6 \times 14539\left(\frac{4}{25}\right)^3 = 87234 \times 0{\cdot}0041 = 357{\cdot}659$.

The distributing girder acts as a girder fixed at both ends; its span is s, and the central moment of strain is $\frac{8{\cdot}04 \times 4 \times 12}{8} = 48{\cdot}24$ inch tons, which must be equalled by the moment of resistance; hence for the distributing girder $\frac{s\,b\,d^2}{6} = 48{\cdot}24$; $\frac{2\,b\,d^2}{3} = 48{\cdot}24$, $\therefore b\,d^2 = 72{\cdot}36$. Also

$m' = b\,d^3 = 357{\cdot}659$. Because $b\,d^3 = b\,d^2 \times d$, $357{\cdot}659 = 72{\cdot}36 \times d$, $\therefore d = \frac{357{\cdot}659}{72{\cdot}36} = 4{\cdot}943'$ inches. Again, $b\,d^2 = b \times (4{\cdot}943)^2 = 72{\cdot}36$; $\therefore b = \frac{72{\cdot}36}{(4{\cdot}943)^2} = 2{\cdot}962$ inches.

With a solid bar of this size there would be prohibitive difficulties in the way of making a satisfactory joint with the cross girders, and if the material be expanded into an open section, the difficulty of obtaining the proper moment of resistance, together with the requisite deflection and suitable sections of metal to build up the distributing girder, will be experienced.

In practice a girder stiffer than that indicated by theory has to be adopted; hence the load on the idle girder is greater than 0·268 W, but it can never exceed 0·3125 W, which would be its value were the distributing girder absolutely rigid; hence by adopting this coefficient perfect safety is secured, and as this is equivalent to twice the weight equally distributed, the saving is $1 - 0{\cdot}625 = 0{\cdot}375$, or 37½ per cent. of the area required by the running load, using a distributing girder.

Many other cases arise in practice in which the distributing or equalising girder is found advantageous; for instance, in bridges carrying ordinary roads, on which the moving load is generally of a uniform character, but is occasionally varied by heavy concentrated loads, such as that presented by a heavy traction-engine or steam-roller. Instead of making each roadway girder heavy and strong enough to carry this load, should it come upon it, by means of a distributing girder, the concentrated load is distributed over several of the ordinary girders.

CHAPTER XII.

PRACTICAL APPLICATION OF FORMULÆ.

HAVING so far elucidated the principles upon which structures should be designed, it seems advisable to show the method of applying them in practice, as although the calculations once explained present in themselves no difficulties, the reproduction of their results in the form of working drawings is not always obvious to the student.

The drawings necessary are: a general plan; side elevation; cross section; longitudinal section; and enlarged views and sections of details. These drawings should be accompanied by a specification stating the quality of materials to be used, and the kind of workmanship to be put into the work. The specification should be very carefully drawn up and *adhered to*, for if the specification is drawn by a competent engineer, there should be nothing in it to require modification *after* the agreement for the execution of the work is signed.

Let a design be required for the superstructure of a railway bridge (the piers or abutments will be treated of under the head of Structures of Stability), to carry a double line of railway, the headway being sufficient to allow a suitable depth to be given to the cross girders, but not enough to allow of the main girders being put under the rails, so that two main side girders will be used, cross girders 4 feet apart between them, combined by a central

distributing girder, and the floor being made of buckled plates. All the girders to be plate girders. The span of the bridge 130 feet.

In order to allow a sufficient clearance on each side of the railway trains, the girders must be 4 feet 6 inches from the outer rail, giving for the clear width between the side girders 25 feet.

To determine the running load, the actual strain produced by a string of locomotives of the heaviest type running on the line should be calculated, and the load per lineal foot capable of producing an equal strain determined for various spans, so as to have the data for any span at hand. For the present case I shall take $1\frac{1}{4}$ tons per line of railway as the running load for the main girders, and 15 tons per pair of driving wheels for running load on the cross girders.

For the large side girders and for the shallower girders the effective depth will be the depth between the centres of gravity of the flange areæ.

The working strains allowed will be—tension, $4\frac{1}{2}$ tons; compression, $3\frac{1}{2}$ tons; shearing, 4 tons per sectional square inch.

In fixing the dead load it is necessary to have some means of estimating the weight of a girder, arch, or chain, to carry any given live load; such weight in the arch to include the spandrels and road girder, and in the chain to include the suspension rods and road girder; the following table will supply these data for carefully designed structures.

If W = weight of girder, or arch, or chain, in tons per foot run; W_2 = total load *on* girder (not including weight of girder); c = coefficient taken from the table; $W = W_2 \times c$.

GIRDERS.

With Flanges of equal Section throughout.	With Flanges proportioned to the Strain.	Span divided by Depth.
·00182	·00145	8
·00198	·00156	9
·00213	·00167	10
·00228	·00178	11
·00243	·00189	12
·00259	·00200	13
·00274	·00211	14
·00289	·00222	15
·00304	·00233	16

ARCHES OR CHAINS.

Rigid Arch.	Chain.	Span divided by Versine.*
·00085	·00073	4
·00093	·00079	5
·00100	·00085	6
·00108	·00091	7
·00117	·00097	8
·00125	·00104	9
·00133	·00110	10
·00141	·00116	11
·001495	·00121	12

The cross girders must first be designed. Each cross girder will carry as dead load its own weight and its share of ballast, floor plates, and permanent way. As running load there will be 15 tons on each pair of rails for every alternate girder: taking the wheel bases of a tank engine at 8 feet, this will be a maximum load for any cross girder, and of this (as shown in the chapter on Combinations of Girders) a part only will act on one cross girder. The top flanges of the side girders for such a span should be made 2 feet 6 inches wide; hence the effective span of the cross girder from web to web will be 27 feet 6 inches, and the load will be placed as shown in Fig. 65. What will

* The versine is the rise of the arch from springing to crown, or the fall of the chain from level of supports in towers to centre.

be the equally distributed load, giving a maximum strain equal to that produced by the 7·5 tons on each rail? Treating these as symmetrical loads, the maximum strain is (using our former notation) $\frac{7{\cdot}5\ (5{\cdot}75 + 10{\cdot}75)}{d} = \frac{123{\cdot}75}{d}$.

The strain from a distributed load is $\frac{W\ l}{8\ d}$ = at the centre;

$\frac{W\ l}{8\ d} = \frac{W \times 27{\cdot}5}{8\ d} = \frac{123{\cdot}75}{d}$; W = 36 tons, and of this the proportionate equally distributed load carried by one girder will be 36 × ·625 = 22·5 tons.

The weight of ¼-inch buckled floor plates, including their joints, strips, and rivets, is 12 lbs. per superficial foot; of ballast 1 foot thick, 120 lbs. per superficial foot; of per-

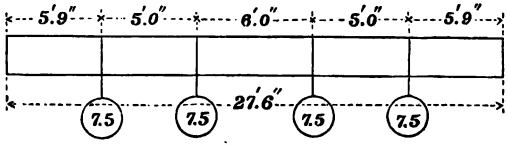

Fig. 65.

manent way, 400 lbs. per yard of double line. The area carried by each cross girder is 27·5 × 4 = 150 square feet; the loads will be—

Plates . . .	110 × 12 =	1320	
Ballast . . .	110 × 120 =	13200	
Permanent Way	1⅓ × 400 =	533	
		15053	lbs. = 6·72 tons;

which, added to 22·5 tons running load, gives 29·22 tons, to which must be added the weight of the cross girder. The girder will be of uniform section, its depth in the centre $\frac{1}{10}$ of its span; hence its weight will be, taking the coefficient from the table, 29·22 × 0·00213 = 0·06224 ton

per lineal foot; the total weight is $0{\cdot}06224 \times 27{\cdot}5 = 1{\cdot}712$ tons, making the total weight to be supported by the cross girder $29{\cdot}22 + 1{\cdot}712 = 30{\cdot}932$ tons, say 31 tons. The maximum strain on either flange will be $\frac{31 \times 27{\cdot}5}{8 \times 2{\cdot}75} = 38{\cdot}75$ tons. The effective depth, being $\frac{1}{10}$ of the span, is 2·75 feet. The area of the top flange in compression will be $\frac{38{\cdot}75}{3{\cdot}5} =$ 11·07 square inches, gross area; the nett area of bottom flange to resist tension will be $\frac{38{\cdot}75}{4{\cdot}5} = 8{\cdot}61$ square inches. For work of this size the flanges will be joined to the web by angle irons 3 inches by 3 inches, by $\frac{1}{2}$ inch thick, the rivets being $\frac{3}{4}$-inch diameter, pitched 4 inches apart from centre to centre. This angle iron rolled full will have an area equal to a bar 6 inches by $\frac{1}{2}$ inch, and in order to get the angle iron of the area calculated upon, it is best to mark it on the drawing by its weight instead of thickness thus: 3″ × 3″ × 10 lbs. per foot, instead of 3″ × 3″ × $\frac{1}{2}$″. After deducting the area of the angle irons, there will be 11·07 — 6 = 5·07 square inches required. For convenience in attaching the buckled floor plates a wide top flange is required; hence the remaining area will be made up by a plate 12 inches wide by $\frac{7}{16}$ inch thick. For the bottom flange nett area is to be taken, as in tension the rivet holes are loss on the section: in each angle iron there are two $\frac{3}{4}$-inch rivet holes, so the nett area of each equals $4{\cdot}5 \times {\cdot}5 = 2{\cdot}25$ square inches, making 4·5 square inches for the two; 8·61 — 4·5 = 4·11 square inches to make up by flange plate; 8$\frac{1}{2}$ inches wide by $\frac{1}{2}$ inch thick will be the nett area required as nearly as it can be met, and to this must be added 1$\frac{1}{4}$ inches width for two rivet holes, making 10 inches by $\frac{1}{2}$ inch. The web should be $\frac{1}{4}$ inch thick; hence the cross section will be as shown in Fig. 66. The effective depth has been taken as 2·75 feet;

that is, between the centres of gravity of the flange areæ. These centres of gravity must be found. If the moments of the areæ of the different parts of a flange be taken about a line parallel to the top of the section, and their sum divided by the total area of the flange, the distance of its centre of gravity from such line will result. For the top flange let the moments be taken about the top boundary *a b*. The moment of each area will be that area multiplied by its mean distance from *a b*. The dimensions of the angle irons are taken full for the rounding at the root *e*.

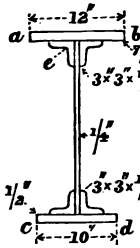

Fig. 66.

One plate	$12 \times \frac{7}{16} \times \frac{1}{2} \times \frac{7}{16}$	$= 1{\cdot}148$
Horizontal limbs of angle irons	$6 \times \frac{1}{2} \times (\frac{7}{16} + \frac{1}{2} \times \frac{1}{2})$	$= 2{\cdot}062$
Vertical ditto	$3 \times 1 \times (\frac{7}{16} + \frac{1}{2} \times 3)$	$= 5{\cdot}812$
		$9{\cdot}022$

The total area is $(12 \times \frac{7}{16}) + 2\,(6 \times \frac{1}{2}) = 11{\cdot}25$ square inches. $\frac{9{\cdot}022}{11{\cdot}25} = 0{\cdot}80$ inch from centre of gravity of top flange to top of girder. For the bottom flange the nett area must be taken about line *c d*.

One plate	$8{\cdot}5 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$	$= 1{\cdot}062$
Horizontal limbs of angle iron	$4{\cdot}5 \times \frac{1}{2} \times (\frac{1}{2} + \frac{1}{2} \times \frac{1}{2})$	$= 1{\cdot}687$
Vertical ditto	$2{\cdot}25 \times 1 \times (\frac{1}{2} + \frac{1}{2} \times 3)$	$= 4{\cdot}500$
		$7{\cdot}249$

The total nett area is $(8{\cdot}5 \times \frac{1}{2}) + 2\,(4{\cdot}5 \times \frac{1}{2}) = 8{\cdot}75$

square inches. $\frac{7{\cdot}249}{8{\cdot}75} = 0{\cdot}83$ inch (nearly) from centre of gravity of flange to bottom of girder. Adding these distances to the effective depth of girder, the total depth becomes 2 feet 9 inches + (0·80 + 0·83) = 2 feet 10·63 inches, so the girder may be made 3 feet deep over all at the centre.

The shearing strain at each end on the web will be half the load, or 15·5 tons, requiring an area of $\frac{15{\cdot}5}{4} = 3{\cdot}875$ square inches; the girder may therefore be tapered down to a depth of 16 inches at the ends. As the flanges are the same throughout, there will be ample strength to allow of the reduction of depth. The shearing strain at any section is equal to the load between that section and the centre; but as the web may be made in two lengths, with the joint at the centre, no joint will come under shearing strain. The angle irons may be made the whole length of the girder. Along the web vertical tee-iron stiffeners should be placed to distribute the load through the web: let one be on each side of the web under each rail, each stiffener being 5 inches wide along the web, and 2½ inches in the other direction; the longest will be nearly 36 inches, and they will all, for convenience of manufacture, be made of the same section; putting two of these back to back, the least width of the pillar so formed will be about 4 inches, giving for the ratio of length to diameter $\frac{36}{4} = 9$. The load over each stiffener is, running load, 7·5 tons; cross-girder ballast and plates over one nearest the centre will be, girder 0·0622 × 5·5 = 0·342 ton: 5·5 feet is the length from the centre of the cross girder to the centre of the two lines of rails. The girders being 4 feet apart, the weight of ballast and plates will be 5·5 × 4 × (120 + 12) = 2904 lbs. = 1·296 tons, and that of the permanent way

133 lbs. = 0·059 ton, making in all 7·5 + 0·342 + 1·296 + 0·059 = 9·197 tons. Taking one-fifth of the breaking weight as working load, it is found from the formula for tee-iron columns that the working strength of the two tee irons is $\frac{1}{5} \times \frac{19}{1 + \frac{81}{900}} = 3{\cdot}5$ tons per sectional square inch: the area required will therefore be $\frac{9{\cdot}197}{3{\cdot}5} = 2{\cdot}62$ square inches. The least thickness to be used for the stiffeners practically will, however, be $\frac{3}{8}$ inch. The load from the stiffeners is passed to the web through rivets; the shearing area required will be $\frac{9{\cdot}197}{4} = 2{\cdot}299$ square inches; each $\frac{3}{4}$ rivet has in section 0·44 square inch, so the number of sections required will be $\frac{2{\cdot}299}{{\cdot}44} = 5{\cdot}2$, that is, 6 sections; each rivet being in double shear, three rivets are necessary; there will, however, necessarily be more than this in the shortest stiffener, so it is evident that in these parts we have ample strength. When the stiffeners are long, as over 2 feet for instance, they should be bent over the angle irons, as shown at A, Fig. 67; but short ones should be put on straight, as shown at B, being packed by the strips shown black in the figure.

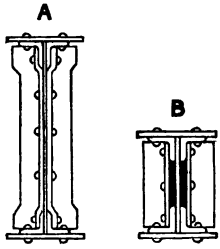

Fig. 67.

The flange plates must each be made in two lengths, and may be joined at the centre. The strength of the top plate is $12 \times 1\frac{1}{8} \times 3{\cdot}5 = 18{\cdot}375$ tons, requiring in rivet area $\frac{18{\cdot}375}{4} = 4{\cdot}593$ square inches. The number of $\frac{3}{4}$ rivets required on each side of the joint $= \frac{4{\cdot}593}{{\cdot}44} = 10{\cdot}4$; that is,

11 rivets. The rivets are in two rows; hence the least we can have will be 6 in each row on each side of the joint, or 12 in the length of the cover-plate; the pitch being 4 inches, the length of this cover-plate will be $12 \times 4 = 48$ inches. Its width and thickness will of course be the same as those of the plates it joins. In the bottom flange, the strength of plate $= 8{\cdot}5 \times \frac{1}{2} \times 4{\cdot}5 = 19{\cdot}125$ tons; rivet area $\frac{19{\cdot}125}{4} = 4{\cdot}78$ square inches; $\frac{4{\cdot}78}{{\cdot}44} = 10{\cdot}8$; that is, 11 rivets as above, so this cover-plate will also be 4 feet long.

There now remains to be determined the rivets for attaching the cross girder to the main side girders, and to the distributing girder. At each end of the cross girder half the effective load will be transmitted to the main girder, the half being 15·5 tons, requiring a shearing area $\frac{15{\cdot}5}{4} = 3{\cdot}875$ square inches $= \frac{3{\cdot}875}{{\cdot}44} = 9$ rivets. In the depth at the ends determined on there will be 4 rivets in shear, but there will be 6 rivets connecting the cross girder with the tee-iron stiffener of the main girder, making 10 in all, and the latter 6 being in tension are stronger than those under shearing stress.

The section of the distributing girder must now be decided. The maximum strain on this will be on the entry of a load on the bridge, when part of it will be in the position of a girder 8 feet span loaded in the centre with $36 \times {\cdot}3125 = 11{\cdot}25$ tons. Let the effective depth of the girder be 8 inches, then the strain on either flange (maximum) $= \frac{11{\cdot}25 \times 8}{4 \times \frac{8}{8}} = 33{\cdot}75$ tons, requiring sectional areæ of flanges, in compression, $\frac{33{\cdot}75}{3{\cdot}5} = 9{\cdot}64$ square inches; in tension, $\frac{33{\cdot}75}{4{\cdot}5} = 7{\cdot}5$ square inches. The section will be made up as shown in Fig. 68, $\frac{3}{4}$ rivets, 4 inches pitch, being

used. Total depth, 10 inches. A is the cross section; B shows the connection of the distributing with the cross girder by 4 rivets on each side. We must see if these will be sufficient. The load to be taken in tension by these rivets is 11·25 tons. $\frac{11{\cdot}25}{4{\cdot}5} = 2{\cdot}5$ square inches; the area in the 8 rivets is $8 \times {\cdot}44 = 3{\cdot}52$ square inches. *a a* and *a′ a′* are the top and bottom cover-plates of the cross-girder flanges. The distributing girder may be made, the angle irons in 30-feet lengths and the plates in 16-feet lengths, so that the joints of the latter fall *between* the cross girders. The covers for the angle irons will require on each side of the joint, top flange, $\frac{3 \times 3{\cdot}5}{4 \times {\cdot}44} = 6$ rivets (3 inches area × 3·5 tons working strength in compression, ÷ 4 tons shearing working strength and ·44 square inch area of one rivet); bottom flange $\frac{2{\cdot}25 \times 4{\cdot}5}{4 \times {\cdot}44} = 6$ rivets. In each case there will be three in each row, or 6 rivets in the length of the cover; but as the rivets are zigzag in the angle irons, it will be equal to 7 rivets in length, or 28 inches. The rivets required for the top flange cover-plates will be on each side of a joint, $\frac{3{\cdot}75 \times 3{\cdot}5}{4 \times {\cdot}44} = 8$ rivets; that is, 4 in each row, or 8 in the length of the plate, making 32 inches. On the bottom flange will be

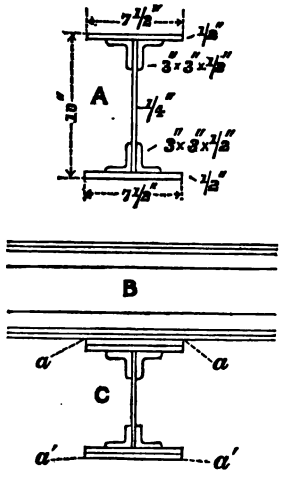

Fig. 68.

required $\frac{3 \times 4{\cdot}5}{4 \times {\cdot}44} = 8$ rivets, making the cover-plate here also 2 feet 8 inches long, which will come in between two cross girders.

It will be observed that in the angle irons in tension I have deducted the rivet holes, as if they came opposite each other instead of zigzag; this is partly because they fall in the thicker part of the metal, and therefore have more than the average thickness out, and partly because the root of the angle iron may be more generally weakened by punching than a flat bar or plate would be.

It must now be seen that the weight of the cross girder and 4 feet of the distributing girder do not together exceed the weight allowed for the cross girder in the calculations. A bar of iron 1 inch square and 1 foot long weighs $3\frac{1}{3}$ lbs.; hence from this can be calculated the weights required. The average sectional area of the web of the cross girder is $\frac{35 + 15}{2} \times \frac{1}{4} = 6{\cdot}25$ square inches; this, added to the area of flanges, gives a weight $= (22{\cdot}25 + 6{\cdot}25) \times 27{\cdot}5 \times {\cdot}3\frac{1}{3} = 2612{\cdot}5$ lbs.; for stiffeners there are 8, each (say) 3 feet long, and in section $5 \times 2\frac{1}{2} \times \frac{3}{8} = 2{\cdot}81$ square inches; their weight $= 24 \times 2{\cdot}81 \times 3\frac{1}{3} = 224{\cdot}8$ lbs. The cover-plates weigh $(4 \times 5{\cdot}25 \times 3\frac{1}{3}) + (4 \times 5 \times 3\frac{1}{3}) = 137$ lbs. The 4 feet of distributing girder weighs $4 \times 21{\cdot}75 \times 3\frac{1}{3} = 290$ lbs.; and for covers (only one occurring in one interval), $2\frac{3}{4} \times 3{\cdot}75 \times 3\frac{1}{3} = 33$ lbs.; taking the sum of these weights, with the addition of 5 per cent. for rivets, we have 3462 lbs. $= 1{\cdot}54$ tons. The weight allowed was 1·712 tons, so there is a little margin here.

The calculations for the main girders can now be proceeded with. In the first place, the total load coming on the main girders is to be summed up. There will be 32 cross girders in the 130-feet span; hence the loads will be as below:—

32 cross girders and distributing girder, $1{\cdot}54 \times 32$ . . . . . = 49·28 tons.

Ballast and plates, $130 \times 27{\cdot}5 \times \frac{132\cdot}{2240}$ = 194·69 ,,

Permanent way, $\frac{130}{3} \times \frac{400}{2240}$ . . . = 7·74 ,,

Running load on 2 lines of railway, $130 \times 2{\cdot}5$ . . . . . = 325·00 ,,

2)576·71 ,,

On each main girder . . 288·35 ,,

These girders will have their plates proportioned to the varying strains, so as to have, under the maximum strain, uniform strain per sectional square inch; the weight of girder to carry this load will therefore be (see table preceding), if the depth = $\frac{1}{12}$th the span,

$288{\cdot}35 \times 0{\cdot}00189 = 0{\cdot}555$ tons per foot. $0{\cdot}555 \times 130 =$ 72·15 tons per girder.

Thus the whole load to be calculated on for each main girder is 288·35 + 72·15 = 360·5 tons. The maximum central strain will be $\frac{360{\cdot}5 \times 130}{8 \times 10{\cdot}83} = 540{\cdot}75$ tons; where 10 feet 10 inches is the effective depth; w = load per lineal foot, equal 2·77 tons.

The gross area of the top flange at the centre will be $\frac{540{\cdot}75}{3{\cdot}5} = 155$ square inches, taking the areæ to the nearest unit; the nett area of the bottom flange will be $\frac{540{\cdot}75}{4{\cdot}5}$ = 120 square inches. The rivets for this work will require to be 1 inch diameter and 4 inches pitch. The central section of the girder may be made up as shown in Fig. 69.

The four angle irons are each 5 inches by 5 inches, by $\frac{3}{4}$ inch thick, with an area of $7\frac{1}{2}$ square inches. There are six plates, five being $\frac{3}{4}$ inch thick, and the outside one $\frac{1}{2}$ inch thick, for the top flange.

There will be four rows of rivets. The bottom flange may be made up of two angle irons 6 inches by 6 inches, by $\frac{3}{4}$ inch thick, four $\frac{3}{4}$-inch plates, and two $\frac{1}{2}$-inch plates. In these large angle irons the zigzag of the rivets can be taken into account in determining nett area; only one hole is deducted in each angle iron. The areæ will now be, dropping the fractions—

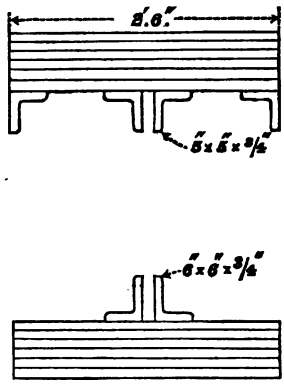

Fig. 69.

For top flange.

4 Angle irons	$5'' \times 5'' \times \frac{3}{4}''$	= 30	square inches.
5 Plates	$30'' \times \frac{3}{4}''$	= 112	,,
1 ,,	$30'' \times \frac{1}{2}''$	= 15	,,
Total sectional area . .		157	,,

For bottom flange.

2 Angle irons	$(6'' - 1'') \times 6'' \times \frac{3}{4}$	= 16	square inches.
4 Plates	$(30'' - 4'') \times \frac{3}{4}''$	= 78	,,
2 ,,	$(30'' - 4'') \times \frac{1}{2}''$	= 26	,,
Total nett area		120	,,

The reduction of the area in proportion to the strain must now be considered. It is evident that this reduction

must be made to the extent of one plate at a time; hence we must find at what points the plates may terminate. As from the ordinary formula the strain and area at any point can be determined, so by inverting the formula the point corresponding to a given sectional area can be found. The formula for strain at any point is $S = \frac{w x^2}{2 d} - \frac{w l x}{2 d}$. Let a = area at any point in square inches, then for top flange $a = \frac{S}{3{\cdot}5}$, and $S = 3{\cdot}5\, a$, $\therefore a = \frac{w}{7 d}(x^2 - l x)$; $x^2 - l x = \frac{7 a d}{w}$; but $x^2 - l x$ will give a minus quantity, or $x^2 - l x = -\frac{7 a d}{w}$. Adding $\frac{l^2}{4}$ to each side of the equation, $x^2 - l x + \frac{l^2}{4} = \frac{l^2}{4} - \frac{7 a d}{w}$. $x - \frac{l}{2}$ is the square root of $x^2 - l x + \frac{l^2}{4}$, therefore $x - \frac{l}{2} = \pm \sqrt{\frac{l^2}{4} - \frac{7 a d}{w}}$. The double sign $\pm$ is used, as the square root may be either plus or minus, for like signs multiplied together make plus; thus $v \times v = v^2$, also $-v \times -v = v^2$. To proceed, $x = \frac{l}{2} \pm \sqrt{\frac{l^2}{4} - \frac{7 a d}{w}}$ for the top flange.

It is evident that $\sqrt{\frac{l^2}{4} - \frac{7 a d}{w}}$ is the distance from the centre of the span to the point on either side corresponding to area a; hence $2\sqrt{\frac{l^2}{4} - \frac{7 a d}{w}}$ will be the length of plates terminating at those points. Call L this length, then starting at the centre with 157 inches, when the top plates stop the area remaining will be $157 - (30'' \times \frac{1}{2}) = 142$ square inches; therefore for the top row of plates, $L_1 = 2\sqrt{\frac{l^2}{4} - \frac{7 a d}{w}} = 2\sqrt{4225 - 27{\cdot}38\,.\,a} = 37$ feet (dropping

fractions), which will be the total length of the top row of plates. Deducting the sections of the other plates in rotation, we get the length of each row; thus $142 - 22{\cdot}5 = 119{\cdot}5 = a_2$; $L_2 = 2\sqrt{4225 - 27{\cdot}38\ a_2} = 62$ feet; $119{\cdot}5 - 22{\cdot}5 = 95 = a_3$; $L_3 = 2\sqrt{4225 - 27{\cdot}38\ a_3} = 80$ feet; $95 - 22{\cdot}5 = 72{\cdot}5 = a_4$; $L_4 = 2\sqrt{4225 - 27{\cdot}38\ .\ a_4} = 95$ feet; $72{\cdot}5 - 22{\cdot}5 = 50 = a_5$; $L_5 = 2\sqrt{4225 - 27{\cdot}38\ a_5} = 107$ feet. This brings us down to the last plate, which must, for the continuity of the structure, be carried the whole length of the girder; we may, however, reduce the angle irons by $\frac{1}{4}$ inch in their thickness; $50 - 10 = 40 = a_6$. $L_6 = 2\sqrt{4225 - 27{\cdot}38\ .\ a} = 112$ feet for the full section of the angle irons. It is hardly worth while, however, to alter the section for the small length remaining.

For the bottom flange, instead of $\frac{7\,a\,d}{w}$, we shall have $\frac{9\,a\,d}{w}$, and $L = 2\sqrt{4225 - 35{\cdot}2\ .\ a}$. Beginning at the bottom plate, $120 - 13 = 107 = a$; $L_1 = 2\sqrt{4225 - 35{\cdot}2\ .\ a} = 43$ feet; $107 - 13 = 94 = a_2$; $L_2 = 2\sqrt{4225 - 35{\cdot}2\ a_2} = 61$ feet; $94 - 19{\cdot}5 = 74{\cdot}5 = a_3$; $L_3 = 2\sqrt{4225 - 35{\cdot}2\ .\ a_3} = 80$ feet; $74{\cdot}5 - 19{\cdot}5 = 55 = a_4$; $L_4 = 2\sqrt{4225 - 35{\cdot}2\ a_4} = 96$ feet; $55 - 19{\cdot}5 = 35{\cdot}5 = a_5$; $L_5 = 2\sqrt{4225 - 35{\cdot}2\ a_5} = 109$ feet. From here the section will run to the ends unaltered.

The lengths of cover-plates must now be determined. Top flange:—The $\frac{1}{2}$-inch plate has a strength of 15 square inches $\times$ 3·5 tons = 52·5 tons. $\frac{52{\cdot}5}{4} = 13{\cdot}125$ square inches of rivet area. The rivets, being 1 inch diameter, have each an area of ·785 square inch; $\frac{13{\cdot}125}{{\cdot}785} = 17$ rivets in each lap. There are four rows of rivets; hence there will be 5 rivets

in each row, and the length of joint-plate on each side of the joint will be 4 inches pitch × 5 = 20 inches. The strength of ¾-inch plate is 22·5 × 3·5 = 78·75; $\frac{78{\cdot}75}{4}$ = 19·69 square inches of rivet area; $\frac{19{\cdot}69}{{\cdot}785}$ = 26 rivets. There will be 7 rivets in a row, equal to 28 inches length on each side of joint. To save in the cover-plates, the joints where possible should be brought together, as shown in Fig. 70, where joints of the five ¾-inch plates are shown following one another; the ½-inch plate has its joint further on. *a b* is the cover, 2 feet 4 inches × 6 = 14 feet long, 2 feet 6 inches wide, and ¾ inch thick.

Angle-iron Covers.—Strength of angle iron 7·5 × 3·5 =

Fig. 70.

26·25 tons; $\frac{26{\cdot}25}{4}$ = 6·56 square inches rivet area; $\frac{6{\cdot}56}{{\cdot}785}$ = 9 rivets, 4 in one row and 5 in the other, making the total length 3 feet 4 inches for both sides of the joint.

Now for the bottom flange. ½-inch plates, strength 13 × 4·5 = 58·5 tons; $\frac{58{\cdot}5}{4}$ = 14·62 square inches rivet area; $\frac{14{\cdot}62}{{\cdot}785}$ = 19 rivets, four rows of 5 rivets each on each side of the joint. ¾-inch plates, strength 19·5 × 4·5 = 87·75 tons; $\frac{87{\cdot}75}{4}$ = 21·94 square inches rivet area; $\frac{21{\cdot}94}{{\cdot}785}$ = 28 rivets, or 7 rivets in each row on each side of joint. Angle irons, strength 8 × 4·5 = 36 tons; $\frac{36}{4}$ tons = 9 square

inches rivet area; $\frac{9}{.785} = 12$ rivets. We shall have on each side of the joint 13 rivets, 7 in one row and 6 in the other.

The ends of the girder will be finished by ½-inch end-plates running down and attached to the web by 3-inch angle irons, ½ inch thick.

In these calculations it is to be remembered that the span is the effective span measured from centre to centre of the bed-plates.

Among the detailed drawings should be furnished a diagram to a distorted scale, showing the distribution of plates in the flanges, with the lengths of all plates and their thicknesses clearly written on, or the lengths, breadths, and weight per lineal foot. The plates should not exceed 20 feet in length, and, in fact, that will be a great length for the plates we are dealing with, but for convenience of making the joints they cannot be much less.

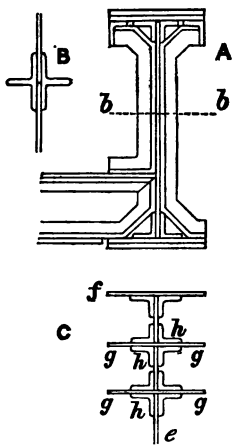

Fig. 71.

The effective depth of the girder has been taken as 10 feet 10 inches; this will bring the depth between the flanges—that is, the depth of the web—to 10 feet 8 inches. This web will be of plates 10 feet 8 inches long, 4 feet wide, jointed by tee-iron stiffeners on each side, turned round at the top and bottom to stiffen the flanges and connect the cross girders with the main girders, as shown in Fig. 71, in cross section of main girder at A. B is a horizontal section at *b b*.

At the ends, where the load is delivered on to the places of support, the web must be strengthened by stiffeners, so

as to form end-pillars, as it were, on which the load can be sustained. The whole load carried by the girder is 360·5 tons, or 180·25 tons at each end, requiring in the end-pillar an area to be determined by the formula for columns. Let the stiffening be made by vertical plates attached to the web and flanges by angle irons turned round like the tee irons, and let the horizontal section be as shown at C, where *e* is the web, and *f* the end-plate, *g g* being the plates making up the pillar, attached by angle irons *h h h*.

If a sandstone bed is used upon which to rest the end of the girder, we find that its working strength in the table is ¼ ton per square inch; hence the top area of bedstone will be at least $\frac{180{\cdot}25}{{\cdot}25} = 721$ square inches, or 5 square feet. But beneath this the masonry is jointed; so I should allow in this case 4 feet length of bearing, so as to get 10 feet bearing surface on the bottom of the girder, and for the length of bearing a bed-plate to be described subsequently will be used. The web is subject to stress peculiar to itself; hence I shall not include the web area over the bearing surface in the effective sectional area of the end-pillar. This pillar will be 4 feet by 2 feet 6 inches over all in plan, and 10 feet 8 inches high; hence its height divided by its least diameter is $\frac{10{\cdot}67}{2{\cdot}5} = 4{\cdot}27$. The multiplier for tee irons includes 19 tons as ultimate strength, but this pillar being built up of plates, I shall take 17 tons, and for working value 3·5 tons; then applying the formula, the strength of the pillar per sectional inch is found to be $\frac{3{\cdot}5}{1+\frac{(4{\cdot}27)^2}{900}} = 3{\cdot}43$ tons. The sectional area requisite will therefore be $\frac{180{\cdot}25}{3{\cdot}43} = 52{\cdot}5$ square inches.

The end angle irons contain 6 inches, the end-plate 15 square inches. If the angle irons *h h h* are 3 inches by 3 inches, by $\frac{5}{8}$ inch thick, the eight will contain 18 square inches, leaving $52{\cdot}5 - (6 + 15 + 18) = 13{\cdot}5$ square inches, to be made up by the plates *g*. Let each of these be made 10 inches wide by $\frac{5}{8}$ inch thick; this will give an area of 15 square inches, a slight margin over what is required.

Next, as to the thickness of the web. The shearing area required at the point of support will be $\frac{180{\cdot}25}{4} = 45{\cdot}06$ square inches, corresponding to 0·35 inch thickness of web. The web, however, will be made $\frac{1}{2}$ inch thick at the ends, and reduced to $\frac{5}{8}$ inch at the centre. The middle third will be $\frac{5}{8}$ inch, $\frac{1}{8}$ on each side, $1\frac{7}{8}$ inch, and the ends $\frac{1}{2}$ inch; these divisions coming thereabouts so as to fit the joints. The rivet area required at the end joints is 45·06 square inches $= \frac{45{\cdot}06}{{\cdot}785} = 58$ rivets; but each rivet is in double shear, hence 29 rivets only will be required. At 4-inch pitch this requires a length of joint $29 \times 4 = 116$ inches. The web is 128 inches deep, leaving, if we deduct the angle-iron depths, a length to which to rivet the tee irons equal to $128 - (5 + 6) = 117$ inches.

I will next determine the strength of tee irons necessary to resist the lateral pressure of wind on the main girder, taking this pressure at 50 lbs. per square foot. The point at which the tee irons are most strained is just at the top of the cross girders, 16 inches above the bottom flange, 9 feet 8 inches from the top of the girder. Each stiffener carries a length of 4 feet of girder, and the mean leverage is half 9 feet 8 inches, or 58 inches; hence the moment of strain will be $\frac{9{\cdot}67 \times 4 \times 50 \times 58}{2240} = 50$ inch tons. The stiffener will act as a cantilever, being of the section shown in Fig. 72. I neglect the web in the calculation of

strength, and put the whole work on the stiffener. The ordinary formula for the moment of resistance, $M = \frac{s.\,b.\,d^2}{6}$, will be used, s being taken at 4 tons.

First we will ascertain the moment of the parts resting against the web $a\,a$; leaving the vertical limbs intact, deducting the rivet holes, and taking the thickness all through at $\frac{3}{4}$ inch, the breadth will be $6 - (2 + \cdot75) = 3\cdot25 = b$; $d = 2 \times \frac{3}{4} = 1\cdot5$; $M = \frac{s\,b\,d^2}{6} = \frac{4 \times 3\cdot25 \times (1\cdot5)^2}{6}$ $= 4\cdot875$ inch tons. The strength required in the vertical limbs will be $50 - 4\cdot875 = 45\cdot125$ inch tons. The strength of the vertical limbs together will be $M = 45\cdot125 = \frac{4 \times b \times d^2}{6}$ $= \frac{4 \times \cdot75 \times d^2}{6}$; $\therefore d^2 = 90\cdot25$; $d = \sqrt{90\cdot25} = 9\cdot5$ inches. So each tee iron must be $4\frac{3}{4}$ inches deep to give the required strength. This is not counting at all upon the rigidity of the girder itself. The pressure of wind I have taken is that assigned to a tornado, so should represent the maximum.

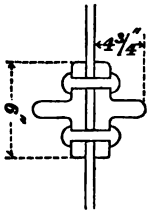

Fig. 72.

Next let us see if the girder is strong enough to resist the lateral pressure, regarding it as held at each end, and carrying the wind as a distributed load.

The total wind pressure, assuming it to blow at an angle to, and act on the whole exposed surface of, both girders, would be 32 tons on the outside girder, and 25·6 on the inside girder, making in all 57·6 tons. In resistance to this force the bridge acts as a girder, of which the floor is the web, and the main girders are the flanges; hence the effective depth is 27 feet 6 inches, and the strain on either girder will be $\frac{57\cdot6 \times 130}{8 \times 27\cdot5} = 34$ tons, a mere nothing on the

flange areæ provided. As to the stability of the superstructure against overturning bodily on its bearings, the wind force by half the depth of the girder gives an overturning moment of $57{\cdot}6 \times 5{\cdot}5 = 316{\cdot}8$ foot tons; the moment of resistance of the bridge (its weight) without any running load, multiplied by half its mean width, is $198 \times 13{\cdot}75 = 2722{\cdot}5$ foot tons, being about nine times the overturning moment.

Having made our girder sufficiently strong to meet all strains, according to the data taken, we must see how its weight accords with that assumed. The weights are easily calculated if we bear in mind that 1 square inch of wrought iron a foot long weighs $3\frac{1}{3}$ lbs., and 1 square foot weighs 10 lbs. for every $\frac{1}{4}$ inch of thickness. I will now take out the weight of one main girder, or rather that part included in the effective span, neglecting fractions:—

ft.			lbs.	lbs.
130·0 × 4 = 520	5″ × 5″	angle irons	@ 25	= 13000
130·0 × 2 = 260	6 × 6	„ „	@ 30	= 7800
67	5 × 5	„ covers	@ 25	= 1675
47	6 × 6	„ „	@ 30	= 1410
800	6 × $4\frac{1}{4}$	tee iron	@ 26·8	= 21440
174	30 × $\frac{1}{4}$	plates	@ 50	= 8700
23	30 × $\frac{1}{4}$	covers	@ 50	= 1150
889	30 × $\frac{7}{16}$	plates	@ 75	= 66675
106	30 × $\frac{7}{16}$	covers	@ 75	= 7950
130	$10\frac{2}{3}$ × $\frac{7}{16}$	web	@ 186·7	= 24271
				154071
		Add for rivet heads 5 per cent .		7703
				161774 = 72·22 tons.

The weight we found, by using the table, was 72·15 tons per girder, so the actual weight comes out with what may be termed an accidental exactness, the difference being inappreciable. So far, then, our design is satisfactory, and drawings may be prepared in accordance with the calculations. The floor plates may be any convenient size, say

3 feet 6 inches by 3 feet, joined by strips 5 inches wide and ½ inch thick; the buckled plates to have a rise of 3 inches, and be ¼ inch thick; rivets in buckled plates to be ⅝-inch diameter, and the plates to be fastened to the cross girders, as shown in Fig. 73. *a a* are the buckled plates fastened to the cross-girder top flange by the ⅝-inch rivets *c*, *c*, thus avoiding any interference with the rivets used to join up the cross girder. The cross-girder cover-plates may be put over the buckled-plate fillets if convenient, the 6″ inches between being made up with packing.

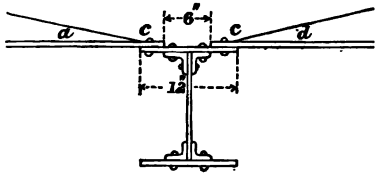

Fig. 73.

The camber to be given to the main girder will be $\frac{130}{40}$ = 3·25 inches; to obtain this the bottom flanges may be made slightly shorter than the top, the web plates being cut to a corresponding taper. From the ordinary formula the radius of the curve taken by the girder is found. If R = radius, l = chord of arc, and v = versine, or rise,

$$R = \frac{l^2}{8v} + \frac{v}{2} = \frac{(130)^2}{8 \times \frac{3{\cdot}25}{12}} + \frac{\frac{3{\cdot}25}{12}}{2} = 7788 \text{ feet nearly;}$$

the ratios of the lengths of the flanges will be as the radii of the bottom and top of the girder. Keeping the bottom flange 130 feet, the top will be $130 \times \frac{7788 + 10{\cdot}66}{7788} =$ 130·18 ft. = 130 ft. 2·16 inches. The extra length of the top flange entails for exact workmanship a slightly longer pitch of rivets, which can easily be got by making proper templates for marking the plates from. These templates are strips of wood drilled where rivet holes are

to come in the plates, these strips being clamped on the plates; the positions of the rivet holes are marked through the holes in the strips by a stump of proper diameter dipped in whitelead. On a ten-foot strip for the bottom flange there will be thirty holes, and for the same number in the top flange the length of strip will be 10 ft. 1·45 inches, say 10 ft. 1½ inches, which is to be divided up into thirty.

The expansion and contraction of iron bridges in England

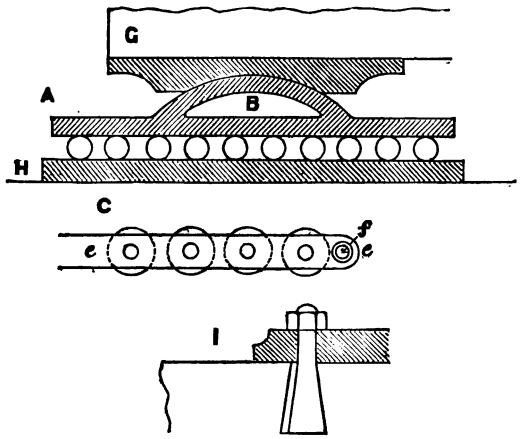

Fig. 74.

have been found to amount to ⅞ inch in 100 feet; hence in the above girders it may reach 0·87 inch. In order to permit free movement longitudinally, one end of each main girder is carried on rollers, which in this case may be 3 inches diameter of wrought iron, between cast-iron plates, of which the bottom one is fixed on the pier, the top one carrying the end of the girder. The rollers have on each side a frame to keep them at their proper distances apart, say 4 inches centre to centre; 10 rollers may be used.

The other end of the girder has plain bearing plates. It is obvious that when a girder deflects its extreme end will be tilted up on the pier, so that it will bear chiefly on the edge of the bed-plate; to avoid this and insure uniform bearing, the bed-plates are formed as shown in Fig. 74, where A is a longitudinal section of the expansion bearing. The end G of the main girder is fastened on a plate capable of oscillating on the lower plate B, which rests on the rollers, carried on the bottom bed-plate H. At C some of the rollers are shown to a larger scale. *e e* is one of the side bars, having notches to receive pins in the ends of the rollers. These side bars are held at their proper distance apart by distance bars, *f*, secured by screw nuts. The top bed-plate is bolted to the bottom of the girder, the bottom one fastened to the bedstone by lewis or rag bolts, of which one is shown at I; these bolts are run in with lead or sulphur, to fix them in the stones.

At the fixed end the bed-plate B rests directly upon the lower one H. In both sets of bearings the plates must be planed, the curved surfaces shaped, and the rollers for the movable end turned accurately to gauge.

The specification must be prepared to accompany the drawings, and in this will be the tests of materials and character of workmanship stipulated for, as well as mode of payment, &c.; the following particulars refer to the former part:—

Cast Iron.—The cast iron to be clean and free from cinder, presenting a grey granular fracture. Its tensile resistance to be not less than 7½ tons per sectional square inch. Test bars to be made from each cast, 3 feet 6 inches long, 1 inch wide, and 2 inches deep; these bars, being supported on bearings 3 feet apart, are not to break under a less central load than 30 cwt., and their deflection under a central load of 6 cwt. is not to exceed ⅛th of an inch.

All columns to be cast vertically, with a head sufficient

to produce a sound casting free from blow-holes and other imperfections, and in no case less than 2 feet in height.

All bolt-holes in the flanges and lugs of cast-iron work to be drilled true to template and to fit the bolts.

Wrought Iron.—All plates, bars, angle, and other wrought iron to break under a strain not less than 22 tons per sectional square inch; its fracture to be fibrous, and free from any appearance of crystalline nature; nor is any lateral separation of the iron to be visible except at the place of fracture. The extension of the iron previous to fracture is not to exceed $\frac{3}{4}$ inch per foot of length, nor to be less than $\frac{1}{2}$ inch per foot. The joints of all plates are to be planed so as to butt truly, the planing being done by a planing machine, and not by a rotary cutter.

All rivets to have ample allowance for making the head, in no case less than $1\frac{1}{2}$ diameters, and the rivets to be solidly closed so as to fill the rivet holes; if collars form at the edges of the rivets, they are *not* to be cut off.

All bolts and screws to be made solid-headed, and the threads, as well as those of the nuts, to be chased with proper chasing tools, and the bodies of the bolts turned; no die-cut threads or ground bolts to be used. The heads and nuts to bear fairly and evenly on the work, and the bolts to be so proportioned that when broken by longitudinal test strain, the fracture shall occur in the body of the bolt, and not by the head pulling off, or the thread of the bolt or nut stripping.

The rivet holes are to be either drilled from the solid, or punched $\frac{1}{8}$ inch less than the size of the finished rivets, and then drilled out to the full size, preferably by a pin drill. All sharp edges round the rivet holes to be taken off.

These few general remarks will show the nature of the stipulations to be made, but the actual amounts of test weights will be varied according to the nature of the work and the locality of manufacture: it is useless to specify

that which cannot be obtained, but care must be taken to calculate the structure for materials of such strength as may reasonably be had, and the specification drawn accordingly, and rigorously enforced.

I have taken this example to illustrate the practical method of calculating structures generally, for I have found students who have acquainted themselves with the theories, at a loss in actually applying them, not clearly seeing how the details are to follow on in proper sequence, or, perhaps, not knowing exactly where to begin; but it is hoped that this one example will serve for all, the general processes being similar, the differences merely occurring in details.

I must now turn briefly to the testing of structures, such as iron bridges, when erected complete. Great care must be taken in the erection to prevent any undue straining or wrenching of any of the parts in lifting, or when in place. A competent man, *acquainted with the character of the strain* for resistance to which the work is designed, should be in charge of the erecting, for a serious and permanent damage may be done to a well-designed and soundly executed structure through the blundering of an ignorant person at this stage, and unfortunately such injury may not be discovered or even suspected until it is made evident by failing, or apparent weakness calls attention to it.

The deflection of a riveted or otherwise built-up structure, in my opinion, is not a conclusive test as to its strength; so much of the rigidity will depend upon the quality of the riveting, that the gross result cannot be regarded as a test of the quality of the material, nor can we determine what to attribute to workmanship, and what to metal. This test is accepted, then, as a negative one, showing, where satisfactory, that the structure is not to be condemned, but not proving that it is perfect. This test, to be of any value, should extend over some time. Directly

a work is completed, and before any load comes upon it, the relative level to some fixed mark should be determined, then the effect of the first load coming on it noted, and the set taken from time to time to observe if there is any increase in the permanent deflection. It should also at intervals be tested to see if a given load passing on to or over it causes always the same deflection; from such tests we can determine whether the work is permanent, or if it be gradually deteriorating. The maximum load should not in the first instance be put on at once, but gradually, so as to allow the joints to take their bearings steadily, and without any sudden shock or jar, which would act like a blow, and so tend unduly to strain or wrench some part of the work.

CHAPTER XIII.

ECONOMICAL PROPORTIONING OF STRUCTURES.

In all structures it is necessary to study economy of cost, as in other matters; hence we must not be contented with merely determining how to proportion our materials to such strains as may occur, but must examine the variations of strain due to different proportions in the main dimensions of the work intrusted to our charge. In many, perhaps the majority of instances, circumstances determine the ratio of depth to span, and other dimensions may also be strictly limited; but in order that they may be applied where possible, the most economical ratios should be determined.

Let us consider the ratio of span to depth of a plate girder. As the depth increases, the strain, and therefore the areæ and weights, of the flanges will decrease: it has, however, been seen that certain requirements of manufacture fix the thicknesses of web in excess of the theoretical thickness, since within certain limits the weight of the web of a girder of given span will increase in direct ratio to the depth of girder. The term web here includes the stiffeners and appendages that practically constitute part of the web structure.

If, then, the flanges diminish as the web increases in weight, and *vice versâ*, it stands to reason that there must be some ratio of depth to span corresponding to a minimum

total weight, and it is for this ratio that we now seek an equation, and in so doing we shall not be restricted to parallel flanged girders, but if necessary make the girder of varying depth.

Let l = span in feet; d = depth in feet; w = load per lineal foot in tons; s = direct resistance of iron in tons per square inch (mean resistance); a = sectional area of both flanges in square inches; a' = vertical sectional area of web (including allowance for stiffeners, &c.) in square inches; t = thickness of web in inches; A = total area of web and flanges.

The weights of the parts will vary directly as their sectional areæ, therefore the ratio giving the minimum total area will also correspond to the minimum total weight. For the area of both flanges the general formula for strain must be multiplied by 2, then at any point distant x from a point of support, $a = \frac{w x^2}{d \cdot s} - \frac{w l x}{d s}$. Also, $a' = 12\ d\ t$; hence $a + a' = \mathrm{A} = \frac{w x^2}{d s} - \frac{w l x}{d s} + 12\ d\ t$. Here we find the positive quantity $\frac{w x^2}{d s} + 12\ d\ t$, and the negative $-\frac{w l x}{d s}$, varying with d, and it is evident that if a point is reached where the value of A is a minimum, and about to change from the descending to the ascending value, making the increment of d very small, the increase of the two qualities must be equal. Let f be an indefinitely small increment of d, then $\mathrm{A}_1 = \frac{w x^2}{s (d + f)} + 12\ t (d + f) - \frac{w l x}{s (d + f)}$. Deducting these from the former values, and equating the differences of the positive and negative quantities, we get—

$$\frac{w x^2}{d \cdot s} - \frac{w x^2}{s (d + f)} + 12\ t\ d - 12\ t\ (d + f) = \frac{w l x}{s (d + f)} - \frac{w l x}{d \cdot s},$$

$\therefore 12\,tf = \frac{w\,x^2}{s}\left(\frac{1}{d} - \frac{1}{d+f}\right) - \frac{w\,l\,x}{s}\left(\frac{1}{d+f} - \frac{1}{d}\right) = \frac{w\,x^2}{s} \times \frac{f}{d^2+df} - \frac{w\,l\,x}{s} \times \frac{f}{d^2+df}$. But as f is taken indefinitely small in relation to d, the value df, as compared with d^2, may be neglected; hence, dividing both sides of the equation by f, we find $12\,t = \frac{w\,x^2}{s\,.\,d^2} - \frac{w\,l\,x}{s\,.\,d^2}$. Multiplying both sides by d, $12\,t\,d = \frac{w\,x^2}{s\,d} - \frac{w\,l\,x}{s\,d}$, or $a' = a$. Hence the sum of the areæ of the flanges will equal the area of the web section when the weight of the whole is a minimum.

From the foregoing equations the value of d at any point may be found, for $12\,t = \frac{w\,x^2}{s\,d^2} - \frac{w\,l\,x}{s\,d^2}$; $\therefore t = \frac{w\,x^2}{12\,.\,s\,.\,d^2} - \frac{w\,l\,x}{12\,.\,s\,.\,d^2}$, and $d^2 = \frac{w\,x^2}{12\,.\,t\,.\,s} - \frac{w\,l\,x}{12\,.\,t\,.\,s} = \frac{w\,.\,x}{12\,.\,t\,.\,s}(x - l)$; $d = \sqrt{\frac{w\,x}{12\,.\,t\,.\,s}(x-l)}$. Changing the sign brings this to a rational quantity without altering the value of the difference between x and l, and $d = \sqrt{\frac{w\,x\,.}{12\,.\,t\,.\,s}(l-x)}$.

A comparison of this equation with those given in any text-book on conic sections will show that the variations of d coincide with the ordinates to a semi-ellipse, of which the span of the girder is the major diameter. Determining then the depth at the centre, and drawing by any ordinary graphic method a semi-ellipse, the most economical form of the girder is obtained; the depth at the centre is found by making $x = \frac{l}{2}$, when $d = \sqrt{\frac{w\,l}{24\,.\,t\,.\,s}\left(l - \frac{l}{2}\right)} = \sqrt{\frac{w\,l^2}{48\,.\,t\,.\,s}}$.

Assuming now that the girder is to be made semi-elliptical in elevation, the formula for the strain at any

point will be found by inserting the value of d; thus, if S = strain on either flange,

$$S = \frac{w x^2}{2 d} - \frac{w l x}{2 d} = \frac{w (x^2 - l x)}{2\sqrt{\frac{w x}{12 . t . s}(x - l)}}, \quad S^2 = \frac{w (x^2 - l x)}{4 \over 12 . t . s}$$

$= 3 w . t . s (x^2 - l x)$, and $S = \sqrt{3 w . t . s (x^2 - l x)}$.

The next point to be considered will be, in the web of the lattice or triangular girder, to determine the most economical angle at which to place the bars.

Taking all the bars to be at the same angle with the horizon, the length of a bar will be the square root of the sum of the squares of the depth, and half the base of a triangle; thus, if b = distance between the apices of two triangles, and d = depth of girder, and L = length of a lattice bar, then $L = \sqrt{\left(\frac{b}{2}\right)^2 + d^2}$. If W = the load on any lattice bar, and S = the strain, $S = W . \frac{L}{d}$. The number of lattice bars will be $\frac{2 l}{b}$, where l is the length of the girder. The sectional area of the lattice bars will vary as the strain; hence the total weight of the web will vary as the strain multiplied by the length of one lattice bar, and by the number of lattice bars, or will vary as $\frac{W L}{d} \times L \times \frac{2 l}{b} = \frac{2 . W L^2 l}{d b}$. Replacing L^2 by its value, we have $\frac{2 W l}{d b}\left(\left(\frac{b}{2}\right)^2 + d^2\right) = 2 W l.\left(\frac{b}{4 d} + \frac{d}{b}\right)$. Of this expression $2 W l$ is for any particular case constant, and it is required to find the relation between b and d that will give a minimum value to $\left(\frac{b}{4 d} + \frac{d}{b}\right)$. Let $b = a d$, a being the value sought, then $\frac{b}{4 d} + \frac{d}{b} = \frac{a d}{4 d} + \frac{d}{a d} = \frac{a}{4} + \frac{1}{a}$. If a be

either increased or diminished when at its minimum by an indefinitely small quantity, the results will be the same, as the value of the whole expression is rising on either side of that particular value of a.

Let the value of a be increased by f for one side of the equation, and diminished by f for the other side, then the two values of $\frac{a}{4}+\frac{1}{a}$ will be $\frac{a+f}{4}+\frac{1}{a+f}=\frac{a-f}{4}+\frac{1}{a-f}$, therefore $\frac{a^2+2fa+f^2+4}{4(a+f)}=\frac{a^2-2fa+f^2+4}{4(a-f)}$; but as f is indefinitely small compared to a, f^2 is so much smaller that it may be neglected; hence $\frac{a^2+2fa+a}{a+f}=\frac{a^2-2fa+4}{a-f}$, $\therefore 2fa^2=8f$, $a^2=4$, $a=2$, and $b=2d$; and if half the base equal the depth, the lattice bars will be at angle of 45 degrees to the horizon, which is, therefore, the most economical angle to use.

In the general arrangement of a structure it is obviously desirable to carry the load on girders passing at once to the points of support, if proper proportions for such girders can be got in; thus, by putting longitudinal girders under the rails of a railway bridge, the weight of the cross girders is saved, and also that part of the sectional area of the main girders required to carry the weight of the cross girders. And similarly a road bridge on longitudinal girders will require less material than if it be carried on cross girders resting upon main girders.

If a bridge carrying a double line of railway, where cross girders are necessarily introduced, be supported by three instead of two main girders, there will be a saving in cross girders, as the strain, and therefore the sectional area, of a girder varies as $\frac{l^2}{d}$, and its weight as $\frac{l^2}{d}\times l=\frac{l^3}{d}$, so that if even the same ratio of depth to span is kept in both cases, the weights vary as l^2. If, however, we use three

girders, the middle one must be kept shallow to allow the traffic the requisite clearance, and this generally limits the depth of the centre girder to 4 feet 6 inches, which corresponds to a span of about 54 feet. For longer spans, the girders being deep, the bridge must be widened, so as to give for each line of railway 14 feet clear width as against 25 feet for the two lines together. Calling the width of the main girders 2 feet, we shall have in one case one cross girder 27 feet long, against, in the other case, two cross girders each 16 feet long, or $(27)^2 = 729$ against $2 \times (16)^2 = 512$, a saving of nearly 30 per cent. on the cross girders.

These principles of economy can only be applied as far as external circumstances will permit of their introduction, and it is easy to conceive cases where economy of construction may be advisedly relinquished to secure other advantages; but, nevertheless, it is highly important to know what are the most economical proportions to adopt.

By methods similar to those shown above, the best proportions of span to height in viaducts, and other ratios between the dimensions of various structures, may with facility be determined.

CHAPTER XIV.

STABILITY.

In contradistinction to those elements that resist the forces to which they are opposed by their strength, stand others which by their masses and stability maintain the positions assigned to them by the designer.

In dealing with the stability of a work, we regard it as a whole or solid mass, and the same assumption occurs in dealing with any component part of it.

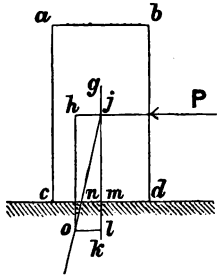

Fig. 75.

There are two ways in which a mass may fail under external force, the first being by overturning upon one of its edges, the second by sliding upon its bed. In Fig. 75 let *a b c d* represent a masonry column 10 feet high and 3 feet square, resting on a flat bed at *c d*. In order to upset, this column must be turned about one of its edges as *c*. Let *g* be the centre of gravity of the mass, from *g* let fall the vertical line *g k*, cutting *c d* in *m*, then the leverage with which the weight of the mass resists the overturning moment is *c m*. This column being symmetrical in form, its centre of gravity will be at its centre of figure, and $c\,m = \frac{1}{2}.\,c\,d$. Let the material of which the column consists weigh

140 lbs. per cubic foot, then the weight of the mass = 140 × 10 × 3 × 3 = 12,600 lbs. If then a force acts at P, say 5 feet from the ground, its value when the mass is on the point of overturning will be $\frac{12600 \times 1{\cdot}5}{5} = 3{,}780$ lbs. The overturning moment and that of stability may also be compared by the parallelogram of forces. If P represents the horizontal pressure against the mass, produce the horizontal line to intersect $g\,k$ in j; make $j\,h$ equal to P, and $j\,l$ equal to the weight of the mass; complete the parallelogram $j\,h\,o\,l$, then $j\,o$ will be the resultant thrust on the pillar; if this resultant fall outside the point c, the mass will overturn; but if it fall at its intersection n between c and d, the mass will be stable. For safety in practice this point is made to fall in the middle third of the base, or the least value of $c\,n = \frac{c\,d}{3}$; but in some works, for the sake of economy of material, this value has been made as low as $\frac{c\,d}{4}$. The harder and stronger the material, the nearer c may the resultant be permitted to pass; but if this point should break off, the base is immediately shortened, and the stability correspondingly reduced.

If the mass is on the point of falling, the resultant will pass through c, and the horizontal force and the weight will be in the same ratio as half the thickness to the height $j\,m$; if we halve the pressure P, the overturning moment will be halved, and the value $c\,n = \frac{c\,d}{4}$; hence this position of the resultant shows a resistance equal to twice the external force. To make $c\,n = \frac{c\,d}{3}$, P must be reduced to one-third of its first value, so the resistance will be three times the external force.

In Fig. 76 $a\,b\,j\,i$ represents a column made of four pieces

of equal mass; the whole must be stable on *i j*, and the superincumbent parts must each be stable on their beds *c d*, *e f*, *g h*, and by drawing the parallelograms of force as shown, it will be found that the stability bears the same ratio to the overturning strain for each part that it does for the whole column.

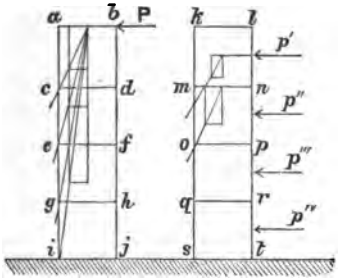

Fig. 76.

k l t s is another column in four pieces, each piece having an equal force *p'*, *p''*, &c., pressing against it: the stability will here be greatest on the top joint *m n*, and gradually decrease to its minimum at *s t*.

Although the stones may not overturn, yet it may happen that one slides upon that beneath it; hence the circumstances under which this may occur must be investigated. This is the resistance of friction, and a margin of resistance in this, as in other respects, must be provided in any structure, to insure durability.

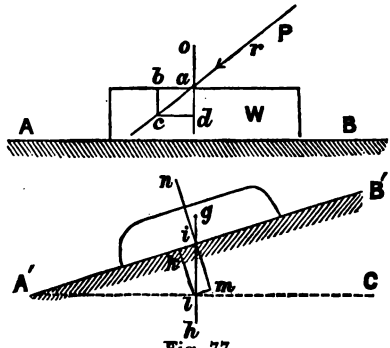

Fig. 77.

Let a mass W, Fig. 77, rest upon a bed or surface A B, and let a force P act upon it in the direction indicated by the arrow. Through the point *a*, where the force P acts upon the mass W, draw a vertical line, and make *a c* equal to P, and complete the parallelogram *a b c d*; then *a d* will

represent the pressure acting at right angles to A B, and $a\,b$ the component acting parallel to it, and tending to slide W upon it. When the inclination of $P\,a$ is such that W is on the point of sliding upon A B, the angle $o\,a\,r$ is called the limiting angle of friction, and $\frac{b\,a}{a\,d}$ is the coefficient of friction, being the ratio giving the horizontal force necessary to slide the mass W under any given pressure acting at right angles to A B.

If no extraneous force is in action, then so long as the bed A B remains horizontal, the only force to which it is subject is the pressure or weight W acting vertically, and therefore at right angles to it; but let the bed be inclined, as shown at A′ B′, to the horizontal A′ C, the weight W will now be resolved into two forces, one acting at right angles to A′ B′, and the other in a direction parallel to A′ B′; the latter tending to cause W to slide down the plane. From g, the centre of gravity of the mass, draw the vertical line $g\,h$, and from i, its point of intersection with A′ B′, mark off $i\,l$ equal to W, and complete the parallelogram $i\,k\,l\,m$; then $i\,k$ is the sliding component of the weight, and $i\,m$ the component pressure on A′ B′; $\frac{i\,k}{k\,m}$ will be the coefficient of friction as before, and if the mass is on the point of slipping, the angle $g\,i\,n$ will be the limiting angle of friction, and the angle B′ A′ C is called the angle of repose, or the natural slope of the material. It will now be shown that the angle of repose is equal to the limiting angle of friction.

A′ $l\,i$ being a right angle, as A′ C is horizontal and $g\,h$ vertical, the angle A′ $i\,l$ is the complement of the angle i A′ l, and $n\,i\,m$ being at right angles to A′ B′, A′ $i\,l$ is the complement to $l\,i\,m$, which is therefore equal to i A′ l; but it is also equal to the limiting angle of friction $g\,i\,n$; hence the angle of repose is equal to the limiting angle of friction.

It is evident, then, that in any structure, the resultant force upon a joint must not make, with a line at right angles to such joint, an angle greater than the limiting angle of friction, or angle of repose, for the material under treatment, and of course must be sufficiently within it to afford a proper margin of safety.

As we can put the joints at any angle we choose, they can always be made so that the resultant thrusts are at right angles to them, or nearly so.

The following table selected from various experiments shows the angle of repose and coefficient of friction for different materials :—

Material.	Coefficients of Friction.	Angle of Repose.
Calcareous Oolite	0·74	36° 30′
Hard „	0·75	36° 53′
Soft Dressed Stones	0·74	36° 30′
Hard „ „	0·75	36° 53′
Common Brick	0·67	33° 50′
Masonry	0·6 to 0·7	31° to 35°
Damp Masonry	0·74	36° 30′
Timber on Stone	0·4	21° 50′
Iron on Stone	0·3 to 0·7	16° 40′ to 35°
„ Metal	0·15 to 0·25	8° 30′ to 14°
Timber	0·2 to 0·5	11° 20′ to 26° 30′
„ on Metal	0·2 to 0·6	11° 20′ to 31°
Masonry on Clay	0·51	27°
Gravel Average	0·84	40°
Dry Sand „	0·78	38°
Sand „	0·40	22°
Vegetable Earth Average . .	0·53	28°
Compact „ „ . .	1·19	50°
Shingle „ . .	0·81	39°
Rubble „ . .	1·00	45°
Dried Clay „ . .	1·00	45°
Wet Clay „ . .	0·29	16°
Dry Powdered Earth . . .	1·03	46°
Dense Compact Earth . . .	1·43	55°

The angles of repose for the earths are those at which these materials will permanently stand, not the angles at

which they first break away, which, for reasons shortly to be shown, are much steeper.

The structures which depend upon their stability for their safety are arches in masonry, abutments, buttresses, retaining walls of various descriptions, sea-walls and break-waters, chimneys, towers, lighthouses, and all structures liable from their localities to be washed away or blown over bodily. Large bridges in exposed and stormy places are at times called upon to resist overturning forces of great magnitude. The general force of the wind is given in the following table:—

Description of Wind.	Velocity in Miles per Hour.	Velocity in Feet per Second.	Force in lbs. per Square Foot.
A hardly perceptible wind	1	1·47	0·005
A pleasant wind	5	7·33	0·125
Brisk gale	10	14·67	0·492
Very brisk	20	29·34	1·968
„	30	44·01	4·429
Very high wind	50	73·35	12·300
Storm	60	88·02	17·710
Hurricane	75	110·00	27·700
Tornado	100	146·66	50·000

The actual measured force of waves has been found to amount to 4,335 lbs. per square foot at Skerryvore, 3,013 lbs. at Bell Rock, and the highest observed 6,000 lbs.

CHAPTER XV.

RETAINING WALLS.

RETAINING walls are of various descriptions, viz. retaining walls for water, also called dams; retaining or revetment walls for earth, either carrying a bank level with the top

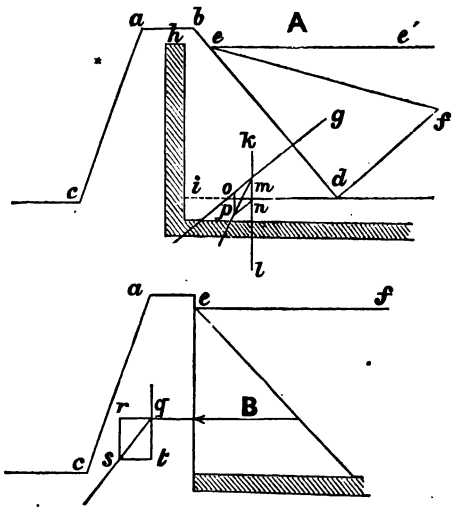

Fig. 78.

of the wall, or surcharged. I will take the retaining wall for water first. In Fig. 78 *c a b d* is the section of a wall for a reservoir side. The shaded part shows a wall of

puddled clay which runs through it to prevent leakage; this puddle is continuous with that running under the bottom of the reservoir. As this puddle wall divides the dam as it were into two walls which are not tied together, hence the pressure of the water must be sustained by one of these walls. If there is no leakage, the inner wall will bear the load; if there is free leakage up to the puddle wall, the outer wall will sustain the pressure of the water. The puddle wall itself is not taken as bearing any part of the load. The working of the first case is shown at A. The pressure of the water will in all cases be at right angles to the surface upon which it presses. For weights a length of wall of 1 foot is taken; also the same thickness of water.

Let $e e'$ be the water level, then draw $d f$ at right angles to $b d$, and equal in length to the vertical depth of water above d; join $e f$, then $e f d$ by 1 foot thick will represent the force of water pressing against the face $e d$. As the area of a triangle is equal to its base multiplied by half its height, and water weighs 62·5 lbs. per cubic foot, then, if the dimensions are taken in feet, the pressure on the wall will be $\frac{e d \times d f}{2} \times 62{\cdot}5 = e d \times d f \times 31{\cdot}25$ lbs. Find g, the centre of gravity of the triangle $e d f$, in the usual way. The sectional area of the wall multiplied by its weight per cubic foot will be the resisting weight. Draw $k l$ through the centre of gravity of the inner wall vertically, and from g draw a straight line at right angles to the face $b d$, and cutting $k l$ in m; make $m o$ equal to the water pressure against the wall, and $m n$ equal to the weight of the section $h i d b$; complete the parallelogram $m n p o$; then the resultant $m p$ should fall within the middle third of the base $d i$; thus the stability of the inner wall has been determined. At B is shown a diagram of the outer part of the wall; the water pressure here is against its vertical face. The water

pressure is represented by $q\,r$, where q is a point in a vertical line $q\,t$ passing through the centre of gravity of this part of the wall. $q\,t$ is the weight of the wall for 1 foot length, $q\,r\,s\,t$ is the parallelogram of forces, and $q\,s$ is the resultant thrust upon the wall, which, when produced, should pass through the middle third of the base.

This case can also be treated by comparing the horizontal and vertical forces in their action producing moments about the outer toe of the wall under consideration. It will be observed that the centre of pressure will be one-third of the depth of the water from the bottom. If D be the depth in feet, the overturning moment will be $M = \frac{62{\cdot}5\,D}{2} \times D \times \frac{D}{3} = 10{\cdot}41\dot{6}\,D^3$. If B = the base of wall in feet, w = width of wall at the top, H = height, and x the distance from the outer toe to a vertical line drawn through the centre of gravity, and 3 the factor of safety, W being = the weight of a cubic foot of the wall, the moments of resistance to overturning will be $= W \times H \times \frac{B + w}{2} \times \frac{x}{3} = \frac{W.\,H.\,x}{6}(B + w) = M = 10{\cdot}41\dot{6}\,D^3$. If, practically, D be considered equal to H, which corresponds to the maximum force upon the wall, we shall have $W.\,x.\,(B + w) = 62{\cdot}5.\,D^2$. Let the whole top of the wall be 6 feet wide, and the puddle wall 1 foot thick, leaving 2·5 feet for the width of the top of each of the walls taken above; let the height be 30 feet, and the weight per cubic foot of the wall 120 lbs. Some formula for the value of x must be obtained. $d\ a\ b\ c$ is a diagram section of one part of the dam; let it be

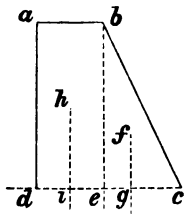

Fig. 78*a*.

divided into two sections for the purposes of calculation, on the horizontal base dc make $de = ab$, and join be; a line drawn through the centre of gravity of $dabe$ vertically will cut de at i, bisecting de; hence the moment of this area about d will be $ad \times ab \times \frac{de}{2}$; but $de = ab$, $\therefore$ the expression becomes $\frac{ad \times ab^2}{2}$. As the centre of gravity of the triangle bec is horizontally one-third of ec from e, and $be = ad$, the moment of the area bec about d will be $\frac{ad \times ec}{2} \times \left(ab + \frac{ec}{3}\right) = \frac{ad \times ec \times ab}{2} + \frac{ad \times \overline{ec^2}}{6}$; and the whole moment is the sum of these, or $\frac{ad \times ab^2}{2} + \frac{ad \times ec \times ab}{2} + \frac{ad \times \overline{ec^2}}{6}$. This quantity, divided by the total area $dabc$, will give the value of x, for the whole moment of the area about d is the area multiplied by x. The whole area is $ad\left(\frac{ab + dc}{2}\right)$; hence

$$x = \frac{2\,(3\,ad \times \overline{ab^2} + 3\,ad \times ec \times ab + ad \times \overline{ec^2})}{6\,ad\,(ab + dc)} = \frac{3\,w^2 + 3\,w \times ec + \overline{ec^2}}{3\,w + 3\,dc} = \frac{3\,w^2 + 3\,w \times ec + \overline{ec^2}}{3\,w + 3\,w + 3\,ec}.$$

$B = w + \overline{ec}$; hence, replacing x and B by their values in the expression $W\,x\,(B + w) = 62{\cdot}5\,D^2$, we get $\frac{62{\cdot}5\ D^2}{W} = \frac{3\,w^2 + 3\,w \times \overline{ec} + \overline{ec^2}}{6\,w + ec}(2\,w + ec) = w^2 + w \times \overline{ec} + \frac{\overline{ec^2}}{3}$

$\therefore\ \overline{ec^2} + 3\,w \times \overline{ec} = \frac{187{\cdot}5.\,D^2}{W} - 3\,w^2$. Adding $\left(\frac{3\,w}{2}\right)^2$ to each side of the equation, and extracting the square roots,

$$\overline{ec^2} + 3\,w \times \overline{ec} + \left(\frac{3\,w}{2}\right)^2 = \frac{187{\cdot}5\ D^2}{W} - 3\,w^2 + \frac{9\,w^2}{4} =$$

$\frac{187{\cdot}5\ D^2}{W} - \frac{3\,w^2}{4}$, and $e\,c = \sqrt{\frac{187{\cdot}5\ D^2}{W} - \frac{3\ w^2}{4}} - \frac{3\ w}{2}$.

This, by filling in the numerical values taken above, becomes $e\,c = \sqrt{\frac{187{\cdot}5 \times 900}{120} - \frac{3 \times 6{\cdot}25}{4}} - \frac{7{\cdot}5}{2} = 33{\cdot}69$ feet. The resistance of the outer part of the wall will be greater, as it will tend to overturn on the edge c, when the leverage becomes $B - x$. In this particular case the value of $x = \frac{3\,w^2 + 3\,w \times e\,c + e\,c^2}{6\,w + 3.\,e\,c} = \frac{3 \times 6{\cdot}25 + 7{\cdot}5 \times 33{\cdot}69 + 1135}{15 + 101}$ $= 12{\cdot}12$ feet, and $B - x = 21{\cdot}57$ feet; hence for the outer wall, $W\,x\,(B + w) = 62{\cdot}5\ D^2$; $W\,x\,(2\,w + e\,c) = 62{\cdot}5\ D^2$, $\therefore e\,c$ $= \frac{62{\cdot}5\ D^2}{W.\,x} - 2\,w = \frac{62{\cdot}5 \times 900}{120 \times 21{\cdot}57} - 5 = 21{\cdot}73$ feet. In the first expression for $e\,c$, the value of $\frac{3\,w^2}{4}$ is so small as to make no practical difference to the root of the whole expression of which it forms a part; omitting it would only alter $e\,c$ to 33·74 feet, and as we do not work to an inch in structures of this description, the small value may be omitted, and the formula, thus simplified, will stand $e\,c =$ $\sqrt{\frac{187{\cdot}5\ D^2}{W}} - \frac{3\,w}{2} = \frac{13{\cdot}7{\cdot}\ D}{\sqrt{W}} - \frac{3\,w}{2}$.

I will now pass to retaining walls for sustaining the pressure of earth, also called revetment walls, though this is more properly a military term.

The conditions under which the pressure of earth acts are widely different from those that have just been occupying our attention; the prism of earth acts as a wedge to push the wall out, and may overturn or push it bodily away; in the latter case it would be too wide to overturn, but too light to resist the lateral pressure.

The earth cannot actually slide without causing considerable friction, both on the back of the retaining wall,

and on the earthen surface upon which it slips; but the former is uncertain in amount, and it may happen that before the slip it does not bear evenly on the wall; hence it is usual in practice to make no allowance for this source of resistance, although the resistance of friction on the bed on which it slides is taken into consideration.

In Fig. 79, let $a\,b$ represent the horizontal surface of a bank, sustained by a retaining wall of which $f\,e\,a\,c$ is the section. If $c\,b$ be the natural slope of the ground, then $a\,c\,b$ is the prism of earth that would ultimately fall away if the bank were not supported; but it does not follow that it will all slip away at once, or that by so doing it would bring a maximum pressure on the back $a\,c$ of the wall. It is necessary to ascertain the line of slip corresponding to the maximum horizontal thrust. Let $c\,h$ be that line, $f\,c\,d$ being the level at which the ground is in front of the retaining wall.

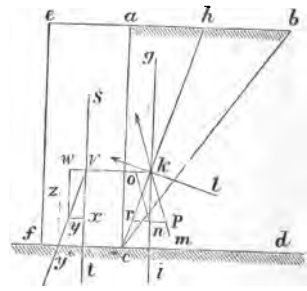

Fig. 79.

Find the centre of gravity of the triangle $a\,c\,h$ at g; from g draw the vertical line $g\,i$, cutting the line of slip $c\,h$ in k; from k draw $k\,l$ at right angles to $c\,h$; now if there is no friction on the line $c\,h$, $l\,k$ would be the direction of the reaction of the bank supporting the prism of earth $a\,c\,h$; but when this prism is *on the point of slipping*, the frictional resistance of the surface comes into action, and alters this direction by an amount equal to the limiting action of friction or angle of repose; therefore make the angle $l\,k\,m$ equal to the angle $b\,c\,d$, then will $m\,k$ be the direction of the reaction of the earth upon the prism $a\,c\,h$. On the vertical line $g\,i$, from the point

of intersection k, mark off $k\,n$, equal to the weight of the prism of earth $a\,c\,h$ (which is, with the wall itself, taken for a length of 1 foot on the face of the wall); complete the parallelogram $k\,o\,n\,p$, then $k\,o$ will be the horizontal thrust against the back $a\,c$ of the wall. It is now necessary to find the position of $c\,h$ that will give a maximum value to $k\,o$ or $n\,p$. Let H = height of wall, w = weight of earth per cubic foot; then the weight of the prism of earth will be $= \mathrm{H} \times \frac{a\,h \times w}{2}$.

Because $g\,i$ is vertical, and therefore parallel to $a\,c$, the angle $n\,k\,c$ = the angle $a\,c\,h$; $k\,l$ is drawn at right angles to $c\,h$; $l\,k\,m$ is drawn equal to $b\,c\,d$; $c\,d$ is horizontal, therefore $a\,c\,d$ is a right angle. As $l\,k\,m = b\,c\,d$, and $n\,k\,c = a\,c\,h$, if $l\,k\,m + n\,k\,c$ be taken from the right angle $c\,k\,l$, and $b\,c\,d + a\,c\,h$ be taken from the right angle $a\,c\,d$, the remaining angle $m\,k\,n$ will equal the remaining angle $h\,c\,b$. Produce the horizontal line $p\,n$ to meet $h\,c$ in r, then the right-angled triangle $k\,n\,r$ is similar to the triangle $c\,a\,h$, and its area varies as that of $c\,a\,h$, and therefore as the weight of the prism of earth $c\,a\,h$; $n\,p = o\,k$. The horizontal thrust will vary as the area of $k\,n\,r \times \frac{p\,n}{k\,n}$. This variable quantity must be examined to ascertain when it, and therefore the thrust, is a maximum. The area of a triangle varies as its base multiplied by its height (the area being base by half the height); hence the quantity with which we have to deal is $= k\,n \times r\,n \times \frac{n\,p}{k\,n} = r\,n \times n\,p$. The ratio of $r\,n$ to $n\,p$, to give a maximum value to their product, is to be determined. If $r\,n$ is increased, $n\,p$ is diminished, and *vice versâ*. When the value is a maximum, then the quantity resulting from diminishing $n\,p$ and increasing $n\,r$ by an indefinitely small quantity, will be equal to that found by increasing $n\,p$ and diminishing $n\,r$ by that same quantity.

Let the indefinitely small quantity be u; then, $(\overline{rn} + u) \times (\overline{np} - u) = (\overline{rn} - u) \times (np + u)$; whence $\overline{np} \times u - \overline{rn} \times u = rn \times u - \overline{np} \times u$, $\therefore np = rn$. Because $knr = knp$, being right angles, and $rn = np$, kn being common to the triangles knr, knp, the angle rkn or $ckn =$ the angle mkn; therefore the equals of these angles, ach and hcb, are also equal, and the line of slip hc bisects the angle acb when the horizontal thrust is a maximum. hc being drawn to bisect the angle acb, and the horizontal thrust determined as previously described, draw st vertically through the centre of gravity of the wall, then produce the horizontal line ko, cutting st in v; make $vw = ko$, and vx = weight of the wall; complete the parallelogram $vwyx$, then vy will be the resultant thrust on the wall, which, when produced, should lie in the middle third of the base. If the resultant cut a joint in y', draw $y'z$ perpendicular to the joint, then the angle $vy'z$ must not exceed the limiting angle of friction for the wall, or what is the same, yz (or ko) divided by vx must not exceed the coefficient of friction, and for safety should not exceed one-half of that coefficient. The angle of the joint can be arranged to suit the resultant thrust, so there need never be any difficulty on this point.

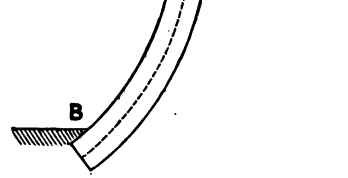

Fig. 79*a*.

Each part of the wall, from the top downwards, must be made stable, as well as the whole wall, so that the strength of the mortar is not called into action. This can be arranged by curving the wall so as to suit the line of thrust, as shown in Fig. 79*a*, where the dotted line is the direction of the thrust, and the wall is curved so as to keep it in the

middle third. The joints, being made square to the curved face, will be about at right angles to the line of thrust.

The values of $a\,h$ for different materials are given in the following table in respect to the height $a\,c$; that is, the actual length $a\,h$ will be $a\,c$ multiplied by the tabular number:—

Material.		$ah = ac \times$
Gravel	Average	·466
Dry Sand	„	·488
Sand	„	·675
Vegetable Earth	„	·601
Compact Earth	„	·364
Shingle	„	·477
Rubble and Dried Clay	„	·414
Wet Clay	„	·734
Powdered Earth, Dry	. . . .	·404
Dense Compact Earth	. . . .	·315

Call the coefficients in the table C; then the weight of the prism of earth $= \frac{H^2 \times C \times w}{2}$. But angle $p\,k\,n$ = angle $n\,k\,c$ = angle $a\,c\,h$; hence $\frac{n\,p}{n\,k} = \frac{a\,h}{a\,c}$; therefore the horizontal thrust $T = \frac{H^2 \times C \times w}{2} \times \frac{a\,h}{a\,c} = \frac{H^3 \times C^2 \times w}{2\,H} = \frac{H^2 \times C^2 \times w}{2}$. This thrust will be found to act at one-third the height from the bottom of the wall; hence the overturning moment is $M = \frac{H^3 \times C^2 \times w}{6}$. If the wall be vertical, and of the same thickness throughout, its weight being W per cubic foot, and thickness in feet t, its moment of resistance to overturning will be $M = (H \times t \times W) \times \frac{t}{2} = \frac{H \times t^2 \times W}{2}$, and if we take as before 3 for the factor of

safety, $\frac{H \times t^2 \times W}{6} = \frac{H^3 \times C^2 \times w}{6}$; $t^2 = \frac{H^2 \times C^2 \times W}{W}$; $t =$ H. C. $\sqrt{\frac{w}{W}}$; and if $w = W$, $t =$ H. C.: thus a plumb wall to support a bank of average compact earth 20 feet in height would require its thickness $t = 20 \times \cdot 364 = 7 \cdot 28$ feet, or 7 feet 3½ inches. When the wall has a battered face, the centre of gravity will not fall over the centre of the base, but nearer the back of the wall, giving a greater leverage to its overturning resistance. The section of the battered wall will be as shown in Fig. 80 by *a b c d*. If this is considered in its elements, the moment of *a c e* about *c* will be its weight multiplied by *c k*, *f g* being a vertical line drawn through the centre of gravity of the triangle *a c e*; $c\,k = \frac{2.\,c\,e}{3}$. The moment of *a e d b*, if *h l* be a vertical line drawn through its centre of gravity, will be its weight multiplied by $e\,i + c\,e$, *e i* being half the thickness *a b*, and *c e* the amount of batter on the face of the wall. Let B = the batter on the face, so, if the batter is 1 to 8, B = ⅛; the back *b d* of the wall being plumb, then the moment of resistance to overturning about the point *c* will be $M = \left(H \times \frac{H \times B}{2} \times \frac{2\,H \times B}{3} + H \times t \times \left[\frac{t^2}{2} + H \times B\right]\right) \times W = W\left(\frac{H^3 \times B^2}{3} + \frac{H \times t^2}{2} + H^2 \times t \times B\right)$. Equating this with the overturning moment, and taking 3 as the factor of safety, $\frac{3\ H^3.\ C^2.\ w}{6} = W.\left(\frac{H^3.\ B^2}{3} + \frac{H.\ t^2}{2} +\right.$

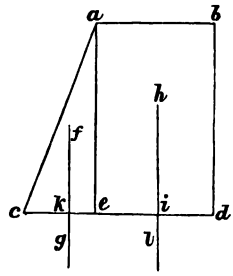

Fig. 80.

$H^2.\ t.\ B\Big)$; whence $t^2 + 2\ H.\ t.\ B = \frac{H^2.\ C^2.\ w}{W} - \frac{2\ H^2.\ B^2}{3}$. Adding $H^2.\ B^2$ to both sides of the equation, and then taking the square roots, $t^2 + 2\ H.\ t.\ B + H.^2 B^2 = \frac{H^2.\ C^2.\ w}{W} - \frac{2\ H^2.\ B^2}{3} + H^2.\ B^2. = \frac{H^2.\ C^2.\ w}{W} - \frac{H^2.\ B^2}{3}$; and $t + H.\ B. = \sqrt{\frac{H^2\ C^2\ w}{W} + \frac{H^2\ B^2}{3}}$; $\therefore\ t = H\sqrt{\frac{C^2\ w}{W} + \frac{B^2}{3}} - H.\ B.$ If 2 be taken as the factor of safety, then $t = H\sqrt{\frac{2.\ C^2.\ w}{3.\ W} + \frac{B^2}{3}} - H.\ B.$ Taking the latter formula, let it be applied to a wall 10 feet high, the weight of the wall per cubic foot being equal to that of the earth, which is to be taken as compact earth, the batter 1 in 8, $t = 10 \times \sqrt{\frac{2 \times \cdot 364^2}{3} + \frac{\cdot 125^2}{3}} - 10 \times \cdot 125 = 1 \cdot 8$ feet thick *at the top;* adding on the batter $\frac{1}{8}$th of the height, the thickness at the ground level becomes $1 \cdot 8 + \frac{10}{8} = 3 \cdot 05$ feet. This formula is commonly used where the ground is not treacherous, but of course the ratios of w to W are filled in when they are not the same. Thus a masonry wall will weigh 130 lbs. per foot, and rammed earth 100 lbs. per cubic foot, which would reduce the above thickness at the top to 1·45 feet.

The foregoing investigations refer to retaining walls carrying banks having a horizontal top surface, and it is interesting to notice that the formula for moment of horizontal thrust will apply to water. The formula arrived at for water was $M = 10 \cdot 416\ D^3$, where $D = H$: that for the overturning moment of earth is $M = \frac{H^3 \times C^2 \times w}{6}$. As water has no angle of repose, C vanishes; $w = 62 \cdot 5$; hence

$M = \frac{H^3 \times 62{\cdot}5}{6} = 10{\cdot}41\dot{6}\ H^3$, the same as the previous expression.

When the wall *a b d c* is loaded, as shown in Fig. 81, it is said to be surcharged, and in this case not only is there a greater load upon the wall, but the centre of pressure acts higher up than one-third its height: in this case it is simplest to determine the horizontal thrust by the graphic method, and then equate its moment with the moment of resistance of the wall.

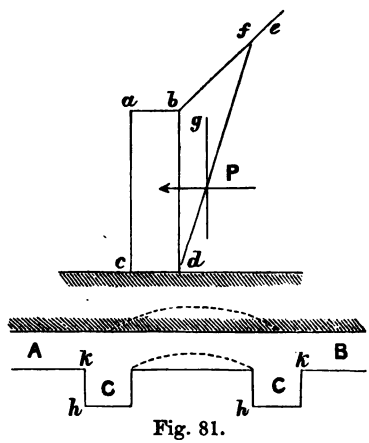

Fig. 81.

Sometimes retaining walls are made with counterforts in front, as shown in the plan A B, where C C are the counterforts; these have the effect of increasing the leverage of the wall, and therefore its moment of resistance, as it must to upset turn over on the edges *h h*, instead of *k k*, *provided* the masonry does not give way at the joint of the counterforts with the wall. In order to strengthen the wall between the counterforts, it is sometimes made arched in plan, as shown by the dotted lines; this will materially assist in preventing it from bulging between the counterforts, and in fact such walls should always be so built.

Counterforts *behind* the wall are not nearly of so much use, as they do not increase the moment of resistance of the other material composing it, though because their own centres of gravity are carried back further from the edge, they are in themselves of more value than the same quan-

tity of material distributed equally on the back of the wall.

Walls requiring a great resistance may sometimes be made hollow, but care must be taken that the weight of material is sufficient to resist the horizontal thrust acting to slide the wall on its base, or these hollow places—pockets or voids, as they are termed—should not be carried completely down through the wall, but left with a sufficient bottom for them to be filled up with earth or concrete, preferably the latter, which by its weight will add to the stability of the structure.

If a retaining wall carry a superposed weight, its stability will be proportionately increased, and this is the case in retaining walls carrying bridges, and also in the abutments of arched bridges. It is a common practice to call the supports of bridges abutments, but this is inaccurate except for arches and certain trusses that abut on and thrust against their supports; in the case of girders the bridge merely puts a weight or vertical pressure on the supports. I therefore avoid applying the term abutments to these supports, piers being a more suitable expression. In arched and trussed bridges the piers are intermediate supports, on which there is, under the full load, only vertical weight, the thrusts from the arches on either side balancing each other.

Retaining walls, when calculated, invariably look heavy both on paper and on the ground, and there is therefore an inclination to cut them down in size: no doubt they form a *very* heavy class of work compared with structures of strength, and it is necessary they should be so, for in making them we are opposing weight to weight, and the resisting mass must bear some proportion to that endeavouring to overthrow it. I call attention to this point especially to caution students against this inclination to diminish the section, for that which is calculated will be

practically requisite, and in no case would I let the factor of safety be *less* than 2. There have been many instances of retaining walls falling down, and more of others which, although they have not actually fallen, have bulged and shown signs of weakness, requiring to be cobbled and patched up either temporarily or permanently, so far as anything connected with such a structure can be said to be permanent. The way in which the temporary support saves the work is by holding it up until the earth behind it, which had been disturbed by building operations, becomes settled down and hardened again, so as to be more in a position to stand by itself than it was when the wall was first completed.

For places where material is of no consequence, *dry* retaining walls are frequently built, having only the coping in mortar. They must be much heavier than mortared walls, and although good enough for small heights, are not to be recommended for large or heavily loaded walls.

CHAPTER XVI.

ARCHES—ABUTMENTS—BUTTRESSES.

Conditions similar to those attending the stability of a retaining wall are imposed in the case of an arch: no part of the arch must turn upon one or other of its edges, and the joints must make a proper angle with the line of thrust, which throughout the length of the arch should lie in the middle third of the depth or thickness of the arch.

As has been shown in a previous part of this work, the tangential force due to a radial force $= w \times r$, where w is the radial force per lineal foot, and r the radius in feet of the element on which it acts, at the point of its action; and this formula will give the horizontal thrust at the crown of an arch, carrying at that point a load w per lineal foot, and having a radius r at the crown: let $p = w\,r$.

In Fig. 82 let the arrow P represent the direction of the force p acting at the crown of the arch. Through the centre of gravity of each arch stone, or voussoir, and its accompanying load, draw a vertical line, as at $e\,f$, $g\,h$. This for each voussoir will pass through the centre of gravity of a mass comprising the voussoir itself, spandrels and internal bearing walls, arches or flags upon the voussoir, also superposed pavement, ballast, and other load.

Produce the direction of the arrow P to intersect $e\,f$ at a, and make $a\,b = p$; mark off $a\ d$ equal to the load on the voussoir D, complete the parallelogram $a\,b\,c\,d$, $a\,c$ will be

the resultant thrust on the first joint; produce this resultant to intersect the second vertical in i, and make $i\,k = a\,c$; make $i\,m$ equal the load on the voussoir E, and complete the parallelogram $i\,k\,l\,m$; $i\,l$ is the resultant thrust on the second joint. By continuing this process the resultant thrust on each joint in succession is found, and finally that on the last joint B, giving the thrust on the abutment T. Through the centre of gravity of the abutment C, draw the vertical line $n\,o$, and produce the resultant T to intersect it at q; make $q\,t = \mathrm{T}$, and $q\,s$ equal the weight of the abutment; complete the parallelogram $q\,t\,u\,s$, then $q\,u$ is the resultant thrust, which should pass through the middle third of the base of the abutment, to give a factor of safety 3: if it only comes within the middle half, the factor of safety is 2.

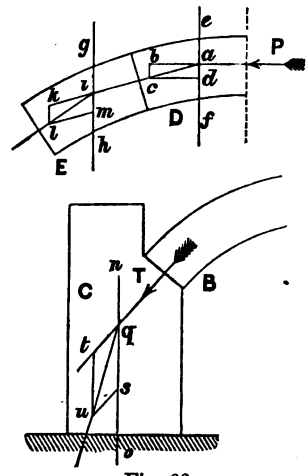

Fig. 82.

It is obvious that if any distribution of load be given, the curve of thrust may be determined as above, and from it the form proper for the arch; and on the other hand, a form of arch being given, the distribution of load necessary to keep the line of thrust in the right course can be determined. In the latter case the parallelograms being drawn to fit the centre line of the given form, the ratios $a\,d$ to $a\,b$, $i\,m$ to $i\,k$, &c., give the proportions that the loads must bear to the thrusts.

As a guide to determining the thickness of the arch at the crown the following formulæ may be adopted:—Let d = depth of keystone in feet, r = radius of arch at crown

in feet; then for a single arch $d = \sqrt{0{\cdot}12.\,r}$, and for one of a series of arches $d = \sqrt{0{\cdot}17.\,r}$. These proportions have been found satisfactory in practice.

If the load is uniformly distributed horizontally over the arch, it will be found that the rise of the arch will be one-fourth of the span.

To determine the amount of the thrust at any point, the horizontal thrust p at the crown must be resolved with the vertical load between the crown and the point at which the thrust is required; the three quantities will then be represented by the three sides of a right-angled triangle as at $a\,b\,c$, where $a\,b$ = horizontal component, $b\,c$ = vertical component, $a\,c$ = resultant thrust; $\overline{a\,c^2} = \overline{a\,b^2} + \overline{b\,c^2}$. Taking w as the uniform load per foot along the arch, horizontally measured, the thrusts will be, at the crown, $w\,r$; at the springing, $\mathrm{T} = \sqrt{(w\,r)^2 + \left(\frac{w\,l}{2}\right)^2}$, if l = span in feet; or (as has been demonstrated in the chapter on Iron Arches), if V = rise of the arch, $\mathrm{P} = \frac{w\,l^2}{8\,\mathrm{V}}$, and $\mathrm{T} = \sqrt{\left(\frac{w\,l^2}{8\,\mathrm{V}}\right)^2 + \left(\frac{w\,l}{2}\right)^2}$; and if $\mathrm{V} = \frac{l}{4}$, $p = \frac{w\,l^2}{8 \times \frac{l}{4}} = \frac{w\,l}{2}$, and $\mathrm{T} = \sqrt{\left(\frac{w\,l}{2}\right)^2 \times \left(\frac{w\,l}{2}\right)^2} = \frac{w\,l}{1{\cdot}414.}$

Resolving the thrust on the abutment vertically and horizontally, the horizontal force will be equal to the thrust at the crown; but acting at the springing, the vertical force is equal to half the weight of the arch: this latter aids the stability of the abutment. If H be the height of the springing from the ground, the overturning moment will be $p \times \mathrm{H} = \frac{w\,l^2}{8\,\mathrm{V}} \times \mathrm{H}$. The moment of resistance to overturning will be for the abutment itself, if its height be h,

and its weight W per cubic foot, $= W \times t \times h \times \frac{t}{2} = \frac{W.h.t^2}{2}$, and the moment of the superincumbent load $= \frac{w.l.t}{2}$; therefore $\frac{w\,l.^2 H}{8.V} = \frac{W h t^2}{2} + \frac{w l t}{2}$.

In determining the stability of massive stone bridges, the weight of the traffic may be ignored in comparison with that of the structure, as it cannot sufficiently vary the line of thrust to practically affect the structure. Suppose an arch of 200 feet radius at the crown is to be built of granite, the thickness of the keystone, that is, the depth of arch at the crown, $= \sqrt{\cdot 12 \times 200} = 4 \cdot 89$ feet, or practically 5 feet. The weight of the stone is 164 lbs. per cubic foot, and $164 \times 5 = 820$ lbs. per foot super of the roadway; upon this there will be about 4 feet more of masonry, ballast, and paving at 140 lbs. per foot, $140 \times 4 = 560$ lbs., making the whole weight at this point 1,360 lbs., while the traffic will not exceed 120 lbs. per square foot, so that its *variations* will not sensibly affect the mass of the structure.

From the peculiar conditions of the case a formula for the general thickness of abutments in relation to the span cannot be evolved, but one including all the varying quantities may be given. From $\frac{w.l.^2 H}{8.V} = \frac{W h t^2}{2} + \frac{w l t}{2}$; it follows that $t^2 + \frac{w l t}{W h} = \frac{w.l^2 H}{4.V.W h}$; adding $\left(\frac{w l}{2 W h}\right)^2$ to each side, and taking the square roots, $t^2 + \frac{w l t}{W h} + \left(\frac{w l}{2 W h}\right)^2 = \frac{w.l.^2 H}{4 V.W h} + \left(\frac{w l}{2 W.h}\right)^2$; $t + \frac{w l}{2 W h} = \sqrt{\frac{w l^2 H}{4.V.W h} + \left(\frac{w l}{2 W.h}\right)}$ and $t = \sqrt{\frac{w l^2 H}{4.V.W.h} + \left(\frac{w l}{2 W.h}\right)^2} - \frac{w l}{2 W h}$. In applying this formula w must be multiplied by the factor of safety.

The abutments of bridges may be strengthened by counterforts placed behind them, as described in connection with retaining walls. Of piers between arches (if the arches be not equal), it is to be observed that their stability must be equal to sustaining the difference of the thrusts from either side.

It is necessary to show that generally there is ample strength to resist the thrust coming upon the materials. I will take a case of an arch in brick, 40 feet span, 28 feet wide over all, 2 feet thick, carrying two lines of railway, the rise of the arch being 10 feet.

Taking the brickwork and the ballast, of which there will be 2 feet in thickness, all at 1 cwt. per cubic foot, the load at the crown will be $1 \times 4 \times 28 = 112$ cwt. The average weight between the crown and the abutment may be taken at 140 cwt. per foot lineal. The load due to the railway may at a maximum be taken at 50 cwt. per lineal foot for the two lines of railway; hence the load w will be 162 cwt. at the crown, and average 190 cwt. along the bridge; the thrust on the abutment will be $T = \sqrt{\left(\frac{w l^2}{8 V}\right)^2 + \left(\frac{w l}{2}\right)^2} = \sqrt{\left(\frac{162 \times 40 \times 40}{8 \times 10}\right)^2 + \left(\frac{190 \times 40}{2}\right)^2}$ = 4,994 cwt. (nearly), or 249·7 tons. The sectional area at the abutment is 28 feet by 2 feet thick, or 56 square feet; hence the thrust is 4·4 tons per sectional square foot. The weakest kind of brick given in our table (in chapter on Building Materials) does not begin to crack under 43·4 tons per square foot, or practically ten times the maximum strain.

In Fig. 83 is shown a buttress used to assist an abutment in resisting the thrust of an arch. The arch, of which half is shown at *a b*, producing a thrust too great for the abutment *b c*, a part of it is carried by the member *f* to the buttress *d e*.

By increasing the height of the spire *d*, the stability of

the buttress *d e* is increased, and at the same time an elegant appearance is given to the work. This kind of construction is to be found principally in church architecture.

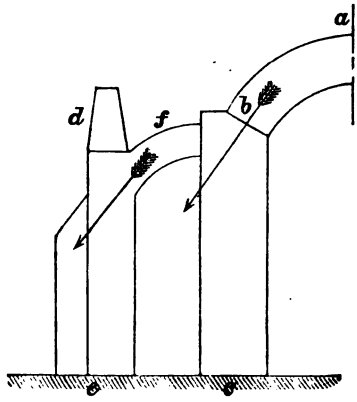

Fig. 83.

In Gothic and other ornamental arches the curve of equilibrium can be determined by means of the parallelogram of forces, similarly to those already investigated.

CHAPTER XVII.

PIERS AND FOUNDATIONS.

The piers supporting a bridge or viaduct have two duties to perform: one to support the weight of the superstructure, the other to resist the attacks of the elements, either the wind on land, or water in rivers, estuaries, and on the seashore.

When coming to the consideration of the forces exerted by the winds and waves, we are unfortunately stepping off that solid ground of fact which we have hitherto been treading, to find ourselves relying upon at best but doubtful bases. Notwithstanding the improved appliances adapted for the scientific investigation of meteorological conditions, the data furnished as to the actual energy of the elemental forces are in a very unsatisfactory state. The feeling must still exist that prompted Smeaton, when justifying his system of joggling the stones in the Eddystone Lighthouse, to write, "Where we have to do with, and to endeavour to control, *those powers of nature that are subject to no calculation*, I trust it will be deemed prudent not to omit in such a case anything that can without difficulty be applied, and that would be likely to add to the security."

It does not seem that the anemometers are to be relied upon, for their indications do not agree with the velocity of wind as shown by the speed of balloons. Of course there are two sides to this question, and it does not follow that

the balloon was throughout its journey in the same current that actuated the anemometers, but these instruments in the same localities also give different readings at the same time, which is certainly confusing.

A pressure of 50 lbs. per superficial foot has been taken as the pressure during a tornado, and this has been stated to be the highest pressure occurring in this country; but, again, people assert that still higher pressures have been recorded.

In examining the effect of the wind upon a structure there is, moreover, the mode of action to be considered. Suppose, for example, the wind to be blowing not steadily, but in short gusts upon the surface opposed to it; if one gust causes the structure to oscillate, and just as it has passed through this oscillation to and fro, and is about to commence another (and smaller one), a second gust strikes it, the energy of this will be added to what remains of the energy of the first, and by the continuation of such action, the amplitude of the oscillations may be so increased as to cause the structure to overturn, although it is quite stable enough to resist the steady, continuous pressure of the same wind at the same intensity. Increasing oscillation is then a more distinct warning than amplitude of oscillation; for so long as the amplitude does not increase, we may consider the work safe, but when it increases regularly, the fate of the work will depend upon the duration of the storm.

If the wind blow against a flat surface at right angles to its course, the whole effective pressure will be the pressure of the wind per square foot multiplied by the area acted on; this will not, however, be the case with a round surface, such as one side of a cylindrical column. In Fig. 84, showing a horizontal section of a cylindrical column, let the wind be assumed to be blowing against it in the direction shown by the arrow, and with a force equal p pounds

per superficial foot. At the point *r* the wind will exert its full force on the surface opposed to it; but take two parts of the surface having their centres *a* and *b* at 45 degrees from *r*, and examine the effects of the wind blowing on these parts; its force will be expended, partly in normal pressure at right angles to the surface of the column, and partly in continuing the motion of its particles, but in a direction at right angles to that pressure. Draw the radii *a c*, *b c*, and at right angles to them the tangents *a d*, *b e*. Equal surfaces being taken, the wind forces will be equal: let *a f* and *b g* represent these forces; complete the parallelograms *a k f l* and *b h g i*, then the forces pressing towards the centre of the column will be represented by *a l* and *h b*.

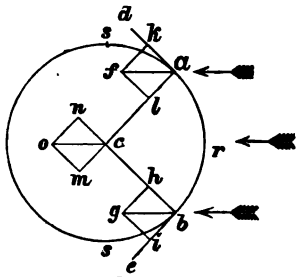

Fig. 84.

Produce the radii *a c* and *b c*, and make *c n*, *c m* respectively equal to *a l*, *b h*. Complete the parallelogram *c n o m*, then the resultant *c o*, which is the resultant pressure of the *two* forces at *a b*, is equal to *one* of them, *a f* or *b g*. The force then in the direction of the wind varies from its maximum at *r* to nothing at *s s*, and it may be shown that the total pressure on the column will be the wind pressure multiplied by the height of the column and by its radius.

In the same manner it may be shown that if the wind blow in the direction of one of the diagonals of a square chimney, the whole pressure upon it will be that due to the wind blowing at right angles to one of its sides, multiplied by ·707.

When the atmosphere is at rest there is the normal pressure of about 15 lbs. per square inch with which it presses upon all matter, and pressing equally on all sides supplies

both the action and reaction; but a wind blowing past an obstacle will rarefy the air behind it, and thus in some degree add atmospheric pressure to the dynamic force of the wind in front of the obstacle.

In dealing with solid works there can now be no difficulty for a given wind force to determine the total pressure on the structure, neither should there be in the case of complex structures, such as braced iron piers and girders, for of course the piers have not only the wind acting directly upon them to resist, but also the wind pressure against the superstructure; but I must here caution the student against regarding one part of a braced structure as sheltering another part from the wind, for a very slight angle will cause the wind to strike on the leeward members of the work, and if the parts are not very close together a direct wind will affect those members, for the shelter given by a narrow obstacle does not extend to any great distance, and also the passing wind tends to draw away the air from behind the first column and crowd it against the next. Assuming the iron pier to be heavy enough for the requirements of stability, the wind action on it must be determined as strain upon the bracing; this has already been done in Chapter V., and we have here only to point out the necessity of making the bracing sufficiently strong for the pier to act against external disturbing forces as a whole, and not, for want of proper connections, go down like a pack of cards.

Masonry piers have a great advantage over iron ones against wind, by reason of their weight, but this very weight sometimes renders them unfit for certain localities where the natural bottom is weak, or where the district has been honeycombed by mining operations, and there iron piers may become a necessity.

I will now take an example of a stone viaduct, taking a centre pier and two half-arches carried by it, and exposed to a wind pressure of 50 lbs. per square foot. Let the arches

be 40 feet span and 28 feet wide, presenting in elevation a semicircular form; pier 80 feet in height, 5 feet thick at the springing, and 9 feet thick at the base. The weight of the two half-arches will be 360 tons, that of the pier 980 tons, making 1,340 tons, which multiplied by half the width at the base—that is, 17 feet—gives as its resisting moment 22,780 foot tons. It may be observed in passing, the weight on the foundation is 4·5 tons per square foot. The centre of gravity of the side surface of the pier is 32 feet above the base, that of the two half-arches, parapets, &c., 95 feet; the surface of the former is 560 square feet, that of the latter 652 square feet. The moment of wind force will be $560 \times 32 \times 50 + 652 \times 95 \times 50 = 3{,}993{,}000$ foot lbs. = 1,782 foot tons, about $\frac{1}{13}$th of the moment of resistance.

I will now consider the stability of an iron pier of similar size. The pier to consist of a group of eight cast-iron columns, each 15 inches in diameter and 1 inch thick, braced together by tee irons 6 inches by 3 inches, by $\frac{1}{2}$ inch thick. The girders to be one under each rail, cross-braced, and covered with iron floor plates and 6 inches of ballast.

The weight of the superstructure, two half-spans, including ballast, will be 56 tons; that of the pier, including bracings and top bearing girders, 88 tons; therefore the moment of resistance to overturning will be $(56 + 88) \times 17 = 2{,}448$ foot tons.

The area of the pier will be eight columns, 1·25 feet diameter, of which being round one-half is taken, $80 \times 1{\cdot}25 \times 4 = 400$ square feet; the bracing 480 square feet; the superstructure, taking the girders, 4 feet deep, 180 square feet. These, being plate girders close together, shelter each other. The wind moment will be $880 \times 32 \times 50 + 180 \times 82{\cdot}5 \times 50 = 2{,}150{,}500$ foot lbs. = 960 foot tons, or $\frac{4}{10}$ths the resistance.

It will be observed that taking in all the columns and bracing the exposed surface is more than that of the solid masonry pier, and some may be of opinion that the wind cannot attack a greater surface than that solid side; however, I consider it injudicious to take less than the surface as estimated above in calculating the stability of iron piers.

If there be not sufficient weight in and on the pier to insure its stability, then the bolts by which it is fastened to the foundations are being strained, and this is a condition which should not be permitted, as a vibratory strain on ordinary holding-down bolts will in time loosen them. However, in all cases of iron piers standing on masonry foundations, the holding-down bolts should be taken through the masonry, and fastened by means of large cotters below massive anchor plates, then there may be some reliance placed upon them.

For piers in tideways and estuaries no general rules can be laid down; the forces to which they will be subjected must be studied, and their intensities ascertained in the localities in which the works are to be executed, and in places where no records are available, experiments should be made, prolonged over a sufficient period and at a proper season to enable the maximum effects to be satisfactorily and conclusively settled before the dimensions of the structure are determined.

In the preparation of foundations it is of paramount importance that the resistance of the ground be uniform, so that in case of settlement, such will occur to an equal extent over the whole area of the foundation; for otherwise the unequal settlement is very likely to cause vertical cracks in the superstructure, and for a similar reason stepped foundations are to be avoided as far as possible, and where they are absolutely necessary, the rise of the steps should be kept as small as is consistent with the general arrangement of the work.

Where the natural subsoil is of a very yielding character, it will be necessary to make a concrete foundation; but even then lateral yielding must be guarded against, and in some places it will be necessary to hold up the foundations by sheet piling, to prevent lateral spreading of the substratum.

In all foundations, where the part of the structure below the natural surface of the ground is relied upon to assist in resisting external overturning efforts, especial care must be used to insure the solidity of the work, and bolts for this purpose should be carried down and fastened beneath anchor plates at the bottom of the foundations. If a pier be partly of iron and partly of masonry, the weakest place will usually be where the ironwork and masonry meet, and this arises from the dissimilitude of the materials in juxtaposition. In order to obviate as far as possible the weakness of such joints, they must be made continuous over the whole area of base. In a masonry foundation carrying a masonry pier, the load comes uniformly upon such foundation; but if the pier is of braced columns, the load is concentrated upon the bases of the columns, and therefore presses only on certain parts of the masonry foundation, and so causes cross strains upon it, tending by transverse rupture to break its continuity, and to render it useless to resist overturning strains. The bases of the columns must be expanded, and fairly secured to a base plate, which is itself connected with the masonry foundation by vertical bolts passing down to an anchor plate at the bottom.

A very convenient foundation for an iron pier is formed by screw piles, which may be used in almost any soil that does not contain boulders; if these are present, the piles are likely to lodge upon them, and then further turning only churns up the soil, leaving the piles without any steady bottom. If the piles require to be in more than one length

below the ground, no external flanges should be used in that part, for the steadiness of the pier will be in great measure dependent upon the close contact of the surrounding earth with the piles, and if external flanges are used, as they are dragged down during the descent of the piles, they must necessarily break up and loosen the soil, which should be firm, and press closely upon the piles.

CHAPTER XVIII.

BUILDING MATERIALS.

It is very necessary that persons in charge of important works, both in regard to design and execution, should be acquainted with the properties of all the materials with which they may be called upon to deal; I shall therefore here briefly describe the various kinds of stone and brick commonly used.

Granites.—Granites generally contain large percentages of silica, the proportion varying from 65 to 80 per cent. Although often very hard and tough in some places, on account of the decomposition of some of their constituents they become soft enough to cut with a spade; discretion must therefore be exercised in their selection, and the natural faces of the rock carefully inspected in the quarries from which it is proposed to obtain the stone. Its specific gravity is about 2·6; a cubic foot weighs 166 lbs.; a cubic yard 2 tons; and the latter contains ordinarily about 3·5 gallons of water, and is capable of taking up about another gallon on immersion in pure water.

The average pressure required to crush various samples has been found to be for the following varieties, in tons per superficial inch, Herm, 6·64; Aberdeen, 4·64; Heytor, 6·19; Dartmoor, 5·48; Peterhead, 4·88; Penryn, 3·45; Ballybeg, 3·17. The stones have, however, cracked respectively at 4·77, 4·13, 3·94, 3·52, 2·88, and 2·58 tons; that for Ballybeg not being recorded.

Limestones.—All limestones are liable to decay, the more rapidly as the stone is less crystalline, under a wet climate, especially when this is accompanied by a smoky gaseous atmosphere. The Mansfield Dolomites contain a considerable proportion of silica, about 50 per cent., and furnish building-stones of good quality. The oolitic limestones are largely used for building purposes; they are of moderate hardness and durability, usually white or pale yellow.

The inferior oolite is a fine-grained, compact freestone, slightly shelly, and yielding blocks of considerable size, soft enough to be cut with a saw when first quarried, but hardening on exposure.

Portland stone is a nearly pure limestone, containing a little silica and magnesia. It is denser than the oolitic limestones, weighing from 135 to 147 lbs. per cubic foot. This stone has, however, lately given place largely to Kentish rag—a calcareous sandstone, light yellow or brown in colour, and sometimes shelly. The cohesive resistance of the Bolsover Magnesian limestone is 8,307 lbs. per square inch, that of the Huddlestone being only 2,531 lbs. The oolites range from Bath Box stone, 1,492 lbs., up to Kelton stone, 2,556 lbs.

Sandstones.—The British sandstones are derived from the Silurian, Devonian, Carboniferous, and New Red Sandstone formations. The former is extremely hard, and therefore well suited for purposes where great strength is required. The sandstones of the Devonian system are of dense structure, and are found to resist the weather successfully. The carboniferous limestones are generally hard and durable, of yellow or greyish tints, and various degrees of coarseness. The millstone grit generally exhibits either massive coarse-grained blocks suitable for foundations, or finer laminated grits suitable for flagstone or paving. The cohesive resistances of various samples was found to be, per square inch, Craigleith, 7,881 lbs.; Darley Dale,

7,100 lbs.; Heddon, 3,976 lbs.; Kenton, 4,970 lbs.; Mansfield, 5,112 lbs. The forces required to crack and to crush various samples of sandstones were found to be, Yorkshire (Cromwell bottom), 2·87 and 3·94 tons per square inch; Craigleith, 1·89 and 2·97 tons; Humbie, 1·69 and 2·06 tons; Whitby, 1·00 and 1·06 tons.

In using stratified stones they should always in the structure of which they form parts lie on their natural beds; that is, in the same relative positions as they occupied in the quarry.

Bricks.—The qualities of bricks vary very widely with different localities and modes of manufacture, but they should in all cases stand the test of prolonged soaking in water without either cracking or becoming spongy, and it is important that they be cleanly formed, so as to take a uniform bearing when laid in course. The following table shows the strengths of various kinds of brick under pressure, from tests made in 1874. The pressures are in tons per superficial foot. The actual areæ experimented upon varied from 32 to 40 square inches.

Description of Brick.	Cracked.	Cracked generally.	Crushed.
Recessed Red Brick, by Morand's Machine	86·8	108·1	161·6
Wire-cut Red Brick, by Porter's Machine	104·5	155·4	228·1
Recessed Red Brick, by Scholefield's Machine	120·9	157·1	192·4
Wire-cut Red Brick, by Johnson's Machine	125·8	177·8	233·0
Red Brick ground in Mortar Mill	143·9	180·3	200·3
„ „ pugged in Pug Mill	92·7	119·7	163·2
„ „ Bulwer's Machine	109·6	157·4	220·4
Recessed Brick, Nottingham Patent Brick Co.	152·1	180·4	204·9
Smith's Red Brick	46·5	57·9	64·2
Cowley's Stock Brick	43·4	54·7	56·6
Burham Gault Brick, Wire-cut	75·6	114·8	114·9
Blue Staffordshire Brick	136·3	210·9	254·7

Beams of stone, natural or artificial, do not seem to follow the laws of resistance to transverse strain by which materials of greater range of elasticity are ruled, nor has their law of resistance, if there be any, yet been elucidated, so experiments on this subject stand in isolation. In experiments on cement floors it has been found that a breaking load seemed to *punch a hole* in the floor.

Portland Cement.—This cement should weigh at least 110 lbs. per imperial striked bushel, and must be ground sufficiently fine to pass through a No. 50 gauge sieve, leaving a residue of not more than 10 per cent. When mixed up neat and immersed in water, it should, after seven days, be capable of resisting a tensile strain of 200 lbs. per square inch. The following table shows the results of experiments on the tensile strength of cement weighing 123 lbs. per bushel:—

Sample.		lbs.			lbs. per square in.
1 week old	Neat cement	363	Half cement and half	Thames sand	160
1 month	„	416	„	„	201
3 „	„	469	„	„	244
6 „	„	523	„	„	284
9 „	„	542	„	„	307
12 „	„	546	„	„	318
24 „	„	589	„	„	351

The resistance of the cement to crushing strain in tons per square inch is, at three months, 1·71; at six months, 2·43; and at nine months, 3·16 tons.

Concrete.—The concrete most generally used is composed of river ballast and Dorking or Barrow lime, or some lime that burns to a buff colour. The proportions vary according to the quality of the materials and the opinion of the user. I have found a very strong concrete for reservoir bottoms made of one part Ellis's Barrow lime (Leicester), and five parts of gravel; but the proportions used in practice vary from four to twelve parts of ballast to one of lime. The

ingredients are sometimes mixed together, slaked as a mixture, and thrown into the foundation from a certain height. Sometimes the ballast is laid on the site of the work, and the lime poured over it like grout; but if this is done the homogeneity of the concrete must be a matter of accident, and therefore there will most likely be great inequalities of bearing strength, a condition to be carefully avoided in the foundations of heavy works. A third method consists in filling the space to be concreted with water, and throwing in the lime and ballast properly mixed. The first of these methods seems the best and most reasonable, as in that we may be sure of uniformity in the compound if the mixing is carefully attended to.

Instead of gravel, Kentish rubble and granite broken small and properly grouted have been very extensively used for carrying buildings of great weight. Furnace dross has also been used for the same purpose. The great disadvantage of using broken stone is that unless it is very carefully laid vacuities are apt to occur, causing a break of continuity, and ramming is to be avoided, as it causes injury to the lower layers already beginning to set, and if the setting of a concrete be disturbed, it cannot be subsequently relied upon as being solid.

In considering the proper quantity of lime to be added to gravel, we must consider the nature of the mass we are producing. It is really a building up of a mass or wall of pebbles cemented together with mortar, that mortar consisting of the sand existing in the gravel and the lime added thereto; it is evident then that the proportion of lime is to be made in regard to the quantity of sand, and not in regard to the quantity of gravel as a whole.

In preparing concrete the lime must be burned, ground, and used hot, and as it has been found by practice and experiment that one of good Dorking lime to three parts, or, if the lime be very strong, four parts by measure of

clean sand makes excellent mortar, this will serve as a generally good proportion. As to the proportion of stones to sand in the gravel, the experience of those who have paid most attention to this highly important matter shows that the stones should be about double by measure of the sand.

In some experiments on concretes, four pits were prepared, and masses of concrete made in them. No. 1 was made of screened stones only, grouted with lime-water; No. 2, four parts stone and one of sand; No. 3, two parts sand and one of stone; and No. 4, two parts stone to one of sand. When sufficiently set a sharpened pole was forced down into them, and the powers of resistance were found to be in the contrary order to the numbers of experiments, and in various other experiments it was invariably found that the best concrete was that made of two parts stone, one sand, and lime sufficient to make a good mortar with the latter.

The importance of care in preparing concrete foundations can scarcely be overrated, for upon them the safety of the structure depends, and once *covered up out of sight*, defects in the concrete cannot be detected until failure of the work indicates their presence. Keen supervision should therefore be exercised over this part of the work, to guard against the substitution of what is practically a dust-heap for a good solid bed of concrete.

The stones in the gravel should be of various sizes, so as to fill all interstices, and the gravel is better in excess than deficiency, for lime alone is no use; it is only in the presence of the sand that mortar is formed. If pit gravel be used, it must be thoroughly washed to remove all dirt and impurities, and *sharp* river sand added as well if necessary.

When the concrete is thrown into the foundation, whatever levelling of surface has to be done must be quickly done, to avoid interfering with it after it has commenced to set. If being put down in only very thin layers, it may be

rolled, the thickness being regulated by proper battens along the edges.

In the sea-wall at the East Cliff, Brighton, the concrete used was made of hydraulic lime, beach shingle, and sand. In very wet foundations, requiring a quick-setting concrete, blue lias lime has been used with success; but this lime is not to be recommended generally, for although it makes a very hard concrete, it is more brittle than Dorking lime concrete, having very little elasticity. Beton is made of 1 part hydraulic mortar to 1½ of angular stones.

Mortars.—Mortar may be made with 1 part lime to 3 or 3½ *clean* sharp river sand, or 1 lime, 2 sand, and 1 blacksmith's ashes. Hydraulic mortar, 1 part blue lias lime to 2½ parts of burnt clay ground together; or 1 blue lias lime, 6 sharp sand; 1 pozzuolana and 1 calcined ironstone. Ordinary mortar should be prepared by first mixing the ingredients dry, and they should then be slaked and tempered with water and well worked, and used fresh. No mortar that has commenced to set should be permitted to be used, and on no account must old mortar be mixed up again with fresh, unless it has previously been re-burnt.

Mortar in setting gradually absorbs carbonic acid from the atmosphere, and so forms a kind of sandstone, containing carbonate of lime, amounting to about 10 per cent. when the mortar is completely set.

The setting of mortar is naturally a very slow process, and the time required for its completion has been put as high as twenty years, and in some cases it has been found that mortar has not set throughout even in the course of centuries; but this has been due to its outer surfaces becoming so indurated as to be impermeable to the atmospheric carbonic acid, without the access of which the setting cannot be completed.

In spreading mortar, all stones and bricks must be fairly and solidly bedded, not merely carried on the edges of a

bed of mortar, for in this case, if there be any weight above, the splitting of such unevenly bedded stone or brick will probably ensue.

As the strength of any structure will be that of the built stone or brick and mortar, it is evident that as little as possible of the weaker material should be introduced; or, in other words, the joints must be made as thin as possible, and the surfaces to be joined should be made as true as is practicable. For this reason also it is necessary, when random rubble masonry—that is, masonry built up of rubble of all sorts of sizes and shapes, not fitted together—is used to carry a heavy weight, that particular attention should be paid to the quality of the mortar or cement employed.

In structures which are liable to settle it is preferable to use mortar, as the cements, by setting very rapidly, do not allow time for the subsidence of the various parts of the work; hence, as the settlement proceeds, cement joints will be apt to crack.

CHAPTER XIX.

EXECUTION OF WORK—OBLIQUE ARCHES.

In the execution of works in iron, the greater part of the skill required is exhibited in the iron-yard; for when the various parts are made to fit, there cannot be much difficulty in the subsequent putting together, which merely requires care and judgment in handling and lifting heavy weights, especially in rough weather, assuming always that the man in charge of the erection understands the nature of the strains the structure is designed to resist, so that he will not let any part be lifted in a manner that may cause undue strain.

On the other hand, in structures of masonry or brickwork, the skilled labour is put in on the site of the work, where a proper system of inspection must be maintained.

In all brick and masonry work the bonding must be so arranged that the structure is as a whole firmly knit together in every direction; and in works subject to great wrenching strains the component blocks should be joined by joggles, or dovetailed pieces of stone. In ordinary work merely sustaining dead weight, such as abutments and walls, there is usually no vertical bond, the courses merely resting upon each other and cohering by the interposed mortar.

A strong transverse bond for brickwork is that in which the courses are alternately laid lengthwise and

crosswise, or, as it is termed, in headers and stretchers, as at A, Fig. 85. B shows an arrangement of headers and stretchers in every course, the headers being shaded. The headers serve to tie the front courses to those behind, and so prevent the wall from splitting in a direction parallel to the face. The joints, whatever kind of bond is used, should be made as thin as is consistent with the proper bedding of the bricks, which should be thoroughly soaked by immersion in water for some time before using;

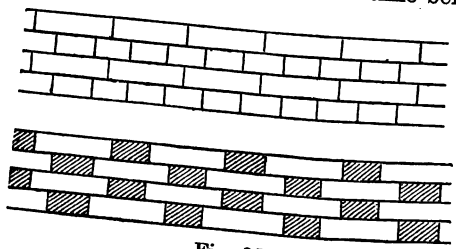

Fig. 85.

the joints also must be completely filled by the mortar, otherwise the bricks will not bear fairly upon each other; and it should always be insisted that the joints shall not exceed a quarter of an inch in thickness. If the bricks are cleanly made, good joints not more than one-eighth of an inch thick can be made; but if the bricks are rough and ill-shaped, thicker joints will follow. A good, sound, well-burnt brick will give a clear ringing sound when struck with a hard substance, but this will not be the case with soft spongy brick.

A greater variety of work will be found in that executed in stone, from random rubble to ashlar; the former being built of blocks of different sizes, laid at random wherever they will fit; the latter consisting of properly squared stones, cut to fit neatly, and accurately joined. Between these two styles there are several intermediate arrange-

ments, such as snecked rubble, where the blocks are not all of the same size, but are placed so as to form a uniform pattern over the face of the work. Block in course, used for parapet and other walls, has all the blocks in each course of the same thickness, though this thickness need not be maintained equal in all the courses. Pierpoints are stones squared up to fixed dimensions, like a brick, but longer in regard to thickness than bricks usually are.

I will now point out the method of working stone into special forms, and also how to determine the correct forms for the stones of certain special structures.

In order to produce a flat bed, in the first place two grooves or drifts are to be cut, one at or near each end of the stone, and these grooves must have their bottoms in the same plane. This can be easily done as follows:— The stone is first placed on a bench or block conveniently placed so that the side to be worked, which is supposed to be the first side, is as nearly horizontal as possible; a rule, strip, or straight-edge is used to test the accuracy of the drifts: this rule has parallel edges, and when the drifts are cut truly horizontal is shown by placing the rule on one edge in them, the upper edge carrying a spirit-level. If the surface of the stone is to be horizontal, three drifts must be cut, the third at right angles to and connecting the first and second, all three running on the same level. These drifts being accurately cut, the intervening stone is cut away until a straight-edge passed over the work touches its surface and the bottoms of the original drifts uniformly all over the bed being worked. One face having been thus accurately dressed, others can be worked from it, always first making driftways from which to gauge the surface as it progresses. It is not always, however, that we require plane surfaces, for the nature of the structure may call for curved beds, and sometimes for twisting or winding beds. The drifts for a curved surface

will require to be cut to special templates attached to stocks, the plane faces of the stones being worked first to give a bearing for the stock. In Fig. 86 A is a stone squared up on three sides; *b* is the stock applied to one side, and carrying the template *c* by which the drift is tested; the edge of the template is indicated by the dotted lines.

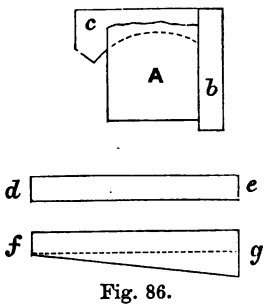

Fig. 86.

When a spiral or twisting bed is required, rules *d e*, *f g* are used. The end *g* of the rule *f g* is deeper than the end *f*, in proportion to the wind of bed required. The edges of *d e* are parallel.

In using these rules to regulate the depths of the drifts, they are connected together by iron links or hooks, so as to preserve a proper distance between them along the bed of the stone. The drifts are then cut so that when the rules are resting in them their upper edges are in the same horizontal plane, as shown by the application of the spirit-level; the stone between the drifts is then cut away, the cutting being regulated by a straight-edge applied at right angles to the first drift, and therefore parallel to the length of the bed, for the first drift is to be cut square to the length of the bed, and the rule with parallel edges placed in that drift.

In an arched bridge all the thrusts will run parallel to the face; hence the coursing joints will run square to these thrusts, and parallel to the axis of the arch or cylinder of which it is a part, if the face of the arch is square to that axis; otherwise the coursing joints, to keep square, or nearly so, to the thrusts, must take a spiral course about the axis; and this is the case in oblique or skew arches.

The correct position of the courses can be graphically determined with facility in a manner that I will now describe.

If a curved surface is supposed to be unrolled and flattened out, the figure thus obtained is called the development of such surface; thus the development of the inside of a semi-cylinder with square ends will be a rectangle, having one side equal to the length of the cylinder —or width of the arch—and that at right angles to it equal to the radius of the cylinder multiplied by 3·1416.

The inner surface of an arch is called its soffit or intrados, and its outer surface the extrados, and upon the developments of these the lines of thrust can be marked and the joints determined; these developments will not be rectangular in an oblique arch.

In Fig. 87 let *a b c* be a section of the arch on the square, showing the intrados only, and let the arch in plan be represented by *a e g f*, the sides *a e*, *c g* being of course drawn parallel to the axis of the cylinder, and at right angles to the chord or diameter *a c*; the angle *a f c*, or *e a f*, is called the angle of skew or obliquity; *a c* is the square span, and *a f* the skew span of the arch; *e n*, drawn at right angles to *e g*, is the width of the arch. Produce *a c* to *d*, making *c d* equal to the length of the arc *a b c*, and divide *c d* into any convenient number of equal parts, as shown, and also divide the arc *a b c* into the same number of equal parts. From

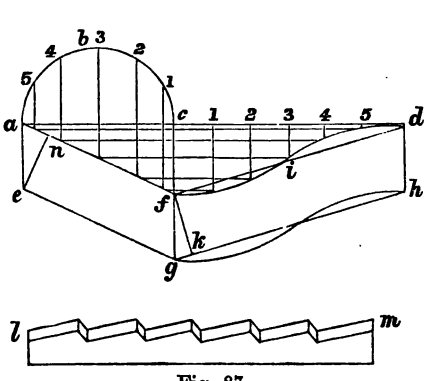

Fig. 87.

the divisions in $c\,d$ draw lines at right angles to it, as shown, and from the divisions of the arc $a\,b\,c$ also draw lines at right angles to $a\,c\,d$, and produce them until they meet the face of the arch in the plan $a\,f$; from these points draw lines parallel to $a\,c\,d$, to meet the vertical lines drawn down from the divisions of $c\,d$, and the points of intersection so found will be points in one edge of the development $f\,g\,h\,d$. It will be observed that these edges are curved, and these curves show the shape that must be given to the lower edges of the face stones.

As I have said above, the strains will be parallel to the face of the arch, hence they will run in curved lines, like $f\,i\,d$, and to these the coursing joints are to be at right angles, or nearly so; these joints will, however, be made straight in development, and at right angles to the straight line made by joining $f\,d$; thus $f\,k$ will be one of the coursing joints, and it will make with the horizontal line of the abutment $f\,g$, an angle $g\,f\,k$, which angle is equal to $c\,d\,f$; for $f\,c\,d$ being a right angle, $c\,f\,d$ is the complement to $f\,d\,c$, and—$d\,f\,k$ being a right angle—it is also a complement to the angle $g\,f\,k$; $c\,f$ is called the obliquity of the arch. If the obliquity of the arch and its rise and square span are given we can calculate the other dimensions, having first determined the length of the arc f. The radius of the arc is, if $l = a\,c$ and $v =$ the rise, $\frac{l^2}{8\,v} + \frac{v}{2}$, a well-known formula based on the properties of the circle: let the span of the arch be 50 feet, and its rise 12 feet, then the radius of the intrados will be $\mathrm{R} = \frac{50^2}{8 \times 12} + \frac{12}{2} = 32{\cdot}04$ feet. The arc being measured in degrees and minutes, its length can be taken from a table of arcs, to be found in most handbooks of mathematical tables; the arc in our example will be 102 degrees, 32 minutes: this will give for the length of the arc 57·28 feet; let the obliquity of the arch be

24 feet: because $a\,c\,f$ is a right-angled triangle, $\overline{a\,f} = \sqrt{\overline{a\,c}^2 + \overline{c\,f}^2} = \sqrt{\overline{50}^2 + \overline{24}^2} = 55{\cdot}46$ feet = skew span of arch. The abutment or impost will be cut into notches, as shown by $l\,m$, these notches having a rise giving angles equal to $g\,f\,k$ or $c\,d\,f$ on the intrados; $c\,d$ being the length of the arc, and $c\,f$ the obliquity, the rise is $\frac{57{\cdot}28}{24}$ = one in 2·39; $a\,n\,e$ is evidently similar to $f\,c\,a$, therefore the length of impost $a\,e$ will bear the same ratio to the width $e\,n$ that the skew span $a\,f$ bears to the square span $a\,c$. Let the width of the arch be 30 feet, then the length of impost measured on the intrados will be $30 \times \frac{55{\cdot}46}{50} = 33{\cdot}278$ feet; let the impost be divided into fifteen notches or checks, the length of each will be 2·227 feet. Draw (Fig. 88) the straight line $a\,b$, from a mark off the 2·39 feet to scale, and erect a perpendicular 1 foot high to c; join $c\,a$, then $c\,a\,b$ = the intradosal angle; make $a\,e = 2{\cdot}227$ feet, the length of one check, and from e draw $e\,d$ at right angles to $a\,c$, then will $a\,d\,e$ be the template for the intradosal notches. It may be made of thin wood or sheet iron, so that it will yield and adapt itself to the curve of the intrados when pressed against it for the lines indicating its boundaries to be marked. The length of the check on the extrados will be of the same length, but its height will be greater in the proportion that the radius of the extrados is greater than that of the intrados; that is, in the ratio of the radius of intrados to that radius plus the thickness of the arch. From the point d draw $d\,f$ at right angles to $a\,b$, then produce it upwards to g, making $g\,f$ = the radius plus thick-

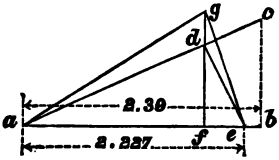

Fig. 88.

ness of arch divided by the radius; join $g\,a$, $g\,e$, then $a\,g\,e$ is the template for the notches on the extrados. Next as to the templates for the face voussoirs. We must determine the elevation of the face of the arch, and ascertain how the face joints occur in it. Let $a\,b\,c$, Fig. 89, be an elevation of the arch square to its axis; $a\,c$ is the line of springing; from a draw $a\,e$, so that if $c\,e$ be at right angles to $a\,c$, the angle $c\,e\,a$ is equal to the angle of skew of the bridge. Through the elevation $a\,b\,c$ draw at right angles to $a\,c$ any convenient number of lines, such as $g\,d$, which produce until they meet $a\,e$ in h; and from h draw $h\,j$ at right angles to $a\,e$, and making $h\,i = d\,f$, and $h\,j = d\,g$. By proceeding in the same manner with the other lines a series of points is obtained, through which the natural elevation of the face can be drawn; its boundaries will be parts of an ellipse. Having got thus the intradosal and extradosal boundaries of the face, the next thing is to find the position of the joints. These may be found by developing the

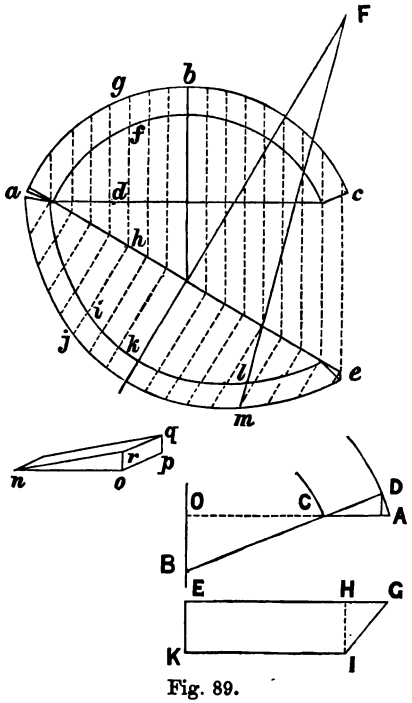

Fig. 89.

intrados and extrados, and marking the courses on them. But the simpler plan is to find the focus or point to which all the face joints converge, which point lies *below* the axis of the cylinder, of which the arch is a part, and in some cases below the periphery of that cylinder altogether. If we suppose the arch to be semicircular on the square, then on the square the joints at the springing will be horizontal. Not so, however, those at the springing on the face, and for this reason: looking at the plan of the springing E G I K, it does not stop square off at H I, but the extradosal spiral passes farther than the intradosal by the quantity H G, and during that passage *it is rising;* therefore at G it is above the springing at I by a quantity equal to the rate of inclination of the extradosal check multiplied by the length H G. We have found the inclination of the intradosal spiral to be 1 in 2·39. Let the thickness of the arch be 2·5 feet; its radius has been found to be 32·04 feet; hence the rate of inclination of the extradosal spiral will be 1 in $2{\cdot}39 \times \frac{32{\cdot}04}{32{\cdot}04 + 2{\cdot}5} = 2{\cdot}217$.

It is evident H G I is the angle of skew; hence H G is to H I, the thickness, as the obliquity is to the square span of the bridge, G I being in the ratio of the skew span. In the elevation of the springing, D A represents the rise from G, A O being a horizontal line passing through the centre O of the cylinder; this line is in the plane of the face of the arch. If D C be the joint at the springing, produce it to meet at B a vertical line O B let fall from the centre O, then B will be the focus of face joints, and O B the excentricity. O B will be to O C (half the span) as A D is to the thickness measured on the face, and therefore as the radius is to the square thickness. $O B = D A \times \frac{R}{t}$, if $t =$ the square thickness; but $D A = \frac{H G}{2{\cdot}217}$, and $H G = t \times \frac{24}{50}$

$= 2{\cdot}5 \times \frac{24}{50} = 1{\cdot}2$; hence $DA = \frac{1{\cdot}2}{2{\cdot}217}$, and $OB = \frac{1{\cdot}2}{2{\cdot}217} \times \frac{32{\cdot}04}{2{\cdot}5} = 6{\cdot}94$ feet. Adding this to the radius, $32{\cdot}04 + 6{\cdot}94 = 38{\cdot}94$ feet, the distance of the focus B from the intrados of the arch at the crown. Let *k* be the centre of the intrados *a i k l*, and *l* the position of a joint at the intrados; draw *k* F at right angles to *a e*, and equal to 38·94 feet; then F will be the point from which all the face joints, as *l m*, will radiate. *n q r o p* is a perspective sketch of the springer.

The formula for the excentricity may be simplified. Let $O = OB$; S = square span; U = obliquity; X = rate of inclination of extrados $\left(\text{as } \frac{1}{2{\cdot}217}\right)$. Because $HG = \frac{U \times t}{S}$, and $DA = HG \times X$, then $O = DA \times \frac{R}{t} = HG \times X \times \frac{R}{t} = \frac{U \times t}{S} \times X \times \frac{R}{t} = \frac{U \times X \times R}{S}$.

If we follow the course of a joint along the intrados it is obvious that it will, commencing from the springing, run out somewhere in the face of the arch, and it is equally obvious that this point must be a face joint. Now it may, and most likely will, happen that the theoretical inclination of the intradosal spiral will not hit a point in the face that will make a joint, and admit of the proper division of the face into equal voussoirs; then we must take the nearest inclination that will fit, and correct our quantities to suit it.

In Fig. 87 *g k* must include an even number of face stones. First find the theoretical value of *g k*; it will be at right angles to *f k*, and will be to *g f*, the length of impost, as *c f*, the obliquity, is to *f d*, which last is called the heading spiral: $fd = \sqrt{\overline{cd}^2 + \overline{cf}^2} = \sqrt{(57{\cdot}28)^2 + (24)^2} = 62{\cdot}1$ feet. Suppose we wish the voussoirs to be about

15 inches thick on the intrados, then $\frac{62{\cdot}1}{1{\cdot}25} = 50$ will be the nearest number; to get a keystone the number must be odd, say 51, then each stone will be 1·217 feet thick.

The theoretical divergence $g\,k = 33{\cdot}278 \times \frac{24}{62{\cdot}1} = 12{\cdot}8$. The nearest number of voussoirs to this quantity is 10, and the actual divergence 12·17 feet; hence the intradosal rate of inclination as corrected is one in $2{\cdot}39 \times \frac{12{\cdot}8}{12{\cdot}17} =$ 2·51 feet.

To this value the other quantities must be corrected. The effect of this practical correction is to slightly alter the angle of the lines of strain to the joints, but this alteration is not of sufficient magnitude to be practically important.

From the methods given above the various templates can be made, and the coursing lines marked on the centring of the arch to guide the builders; the boards covering the centres are called the laggings, and on these the lines are to be laid down. By means of stocks made to a proper angle the stones are worked to the shape of the intrados; but we must show how to determine the dimensions of the twisting rules for working the spiral beds.

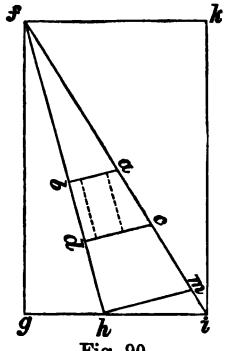

Fig. 90.

In Fig. 90 draw the rectangle $f\,g\,i\,k$ to include the lines $f\,i$ and $f\,h$, drawn to the inclinations of the extradosal and intradosal spirals respectively. Let $b\,d$ be the distance chosen for the masons' rules or strips on the intrados, draw $b\,a\,d\,c$ to represent the rules, then $a\,c$ will be the distance apart of the rules on the extrados, and these distances will vary

as $f\,i$ to $f\,h$. X being the rate of inclination of the extradosal spiral, and Y that of the intradosal spiral, and $f\,g\,h$, $f\,g\,i$ right-angled triangles, $\frac{a\,c}{b\,d} = \frac{f\,i}{f\,h} = \sqrt{1 + X^2} \div \sqrt{1 + Y^2}$; Y as corrected is $\frac{1}{2{\cdot}51}$, and $X = \frac{1}{2{\cdot}32}$; hence $\frac{f\,i}{g\,h} = \sqrt{1 + \left(\frac{1}{2{\cdot}32}\right)^2} \div \sqrt{1 + \left(\frac{1}{2{\cdot}51}\right)^2} = 1{\cdot}023$. If then the distance apart of the rules at $b\,d$ is 24 inches, the distance at $a\,c$ will be $24 \times 1{\cdot}023 = 24{\cdot}552$ inches. The rule $a\,b$ will be made of equal width throughout its length, but $c\,d$ must be wider at the extremity c than it is at d by a quantity now to be found. If $h\,m$ be drawn from h at right angles to $f\,h$, then $h\,m$ will represent the amount of twist corresponding to the length $f\,h$ on the intrados, and the extra width of rule (called T) at c will be to distance apart $b\,d$ as $h\,m$ is to $f\,h$; $T = b\,d \times \frac{h\,m}{f\,h}$. This can be drawn out to a large scale, and the distance measured off to scale.

I have shown that the springers are properly notched to receive the spiral courses of the arch, but it will be seen that the courses of masonry in the abutment below the springers are not laid inclined, but horizontal; hence there is a tendency to slide the skew backs upon the impost; where this is great the work must be fastened together horizontally by iron cramps built into the abutments.

When the skew is 45 degrees or sharper, the edges of the voussoirs on the intrados should be bevelled off for a certain distance, as the sharp arris that would otherwise be left is not likely to endure, being very liable to chip during erection or by subsequent accident.

It is a very common practice to build a brick arch with stone faces, and when this is done care must be taken that the stones are made of the correct thickness to suit the

bricks, and for this purpose the bricks intended to be used must be measured, as they vary in size considerably in different localities, four courses ranging in different parts of the country from 12 to 14 inches, and perhaps more. If the stones are too thick the bricks will get put in with thick joints, and *may* perhaps fall out after the centres are removed, leaving only the face stones standing.

The footings, or bottom courses of masonry or brick structures, should be made of large flat stones (paving or rag stone), so that the bearing may be equally distributed, and the work more thoroughly tied at the extremities. Through stones should also be placed at frequent intervals, say one in every superficial yard of face, to prevent the work from splitting.

In carrying out all descriptions of work a good look-out must be kept for water, and where it appears proper arrangements must be made for carrying it away. It may be brought down the back of a wall by means of a dry lining of broken stone, and then brought through near the bottom by pipes running through the wall or abutment, as is usually done in the case of tunnels: it is desirable, when there is much water thus flowing, to asphalt the back of the work, to prevent the water from working through and injuring the joints.

Masonry arches may be kept water-tight by a layer of good asphalt over the top of the arch and abutments, this being covered by 6 inches of sound puddled clay, which, besides acting to exclude water, serves to protect the asphalt from being cut up by the ballast or road metalling above. Puddle is also used at the bottom of a dry lining to stop and turn the water through the pipes or weep-holes that lead it to the front of the work.

The asphalt should be about 1 inch thick, put on in two layers.

In all these works everything depends upon the quality,

both of materials and workmanship, and as the sharp competition in trade cuts work down to very low prices, if contractors are to get rich it must be at the expense of excellence of work, and this remark applies to work of all descriptions, excluding only such steam and other machinery as will not work if it is not properly constructed.

A structure standing by its stability having no active duty, faults are not detected so long as it does stand, and under the system of using face-work to a great extent, it is not easy to distinguish between work properly built, and that only *heaped up*, after it has once been completed: here, then, is a solid reason for keeping an inspector, and one who is both competent and trustworthy, on works during the whole time of their execution. As a striking example of the difficulty of getting work properly done, I will allude to the almost impossibility of getting the ironfounders in many districts to cast columns vertically; they *will* cast them horizontally, and of course the core floats up, and the columns turn out with a thick side and a thin one, and unless there is a resident inspector the specification becomes a dead letter and a farce. In mentioning this matter another point in connection with it arises, which is that although cast-iron girders do as a rule get tested, the cast-iron columns that carry them do not. The cast iron used to support masonry or brickwork is subjected to very heavy loads, and these, which are the maximum loads, are in many cases *always on* the supports; this is in contradistinction to the moving loads on columns, which probably after their testing in the structure never get a maximum load again; hence the ironwork for the former purpose should be especially sound and always properly tested. The uniformity and homogeneity of a column can be tested and ascertained in the following way:—Let the column be bolted by one end to a solid vertical bed, so that the column is horizontal; note the deflection when

the free end is loaded with the calculated test load; then turn the column one-quarter round, and repeat the test, and so on through the four quarters of the circle. If the column has been cast vertically under a proper head, and is of uniform thickness, the deflection will be the same in each position, otherwise the deflections will indicate the faulty character of the work, even if it does not give way.

In carrying out mixed works of masonry and iron combined, especial thought is called for on the part of the person in charge, for in alterations of detail which may naturally arise in such structures, circumstances may be brought about materially altering the strains on the iron-work, and if it is an architect in charge who has chiefly been accustomed to work wholly in masonry, he may be apt to forget that he is now dealing not only with materials acting by their stability, but also with elements designed only to meet strains and loads of fixed magnitude, in the arrangement of which the enormous factors of safety natural to the masonry as regards its mechanical strength cannot be obtained.

Although the formation of embankments may not be considered as coming properly under the head of "construction," yet if it be not so, the formation referred to has a very important action upon other works properly included under that head, hence may be dealt with in treating of them.

When a bank is to be tipped behind a retaining wall, it should be tipped away from it; that is, the highest level of the bank should, during its progress, always be at the back of the wall, whence it should slope *downwards away from the wall* in each layer: this will prevent the abnormal and unequal pressure caused by making a bank up to or approaching the wall in such a way that its surface is always, whilst in progress, *sloping upwards from the wall.* Where dry lining is used, it should be packed by hand as

the bank rises behind the wall. In running in a bank between two walls, the earth should be carefully tipped, so that the bottom layers are solid and leave no large voids, which may, by the subsequent falling in of superposed earth, cause violent concussions against the side walls.

In concluding these remarks upon the execution of work, I wish to lay stress upon the importance of having ample appliances for the erection of work. It is a very short-sighted policy, in every sense of the phrase, to start with gear insufficient either in quantity or power, leading, as such a course must inevitably do, to vexatious delays, if not to serious accidents.

Under the necessity to get things into place by a given time, the erector may be induced to lift work in a very unsuitable way, merely because he has not the proper tackle for the efficient carrying out of his instructions, and the greatest practical knowledge cannot make up for the lack of machinery.

The student must not, in his early experience on works, take it for granted that all he sees done is done in the best way, but he should carefully watch every detail, and *in his own mind* examine and criticize the operations proceeding around him, and especially note their results in the stability or otherwise of the works in the execution of which they have been used.

CHAPTER XX.

STRENGTH OF MATERIALS.

In order to apply practically the formulæ resulting from processes of reasoning, certain data, obtainable only from experiment, must be furnished to the constructor in the form of the strengths of various materials used in construction.

At first sight it may seem sufficiently easy and simple to obtain from experiment a table of the resistances of materials; but practically it is a matter requiring great care in the conduct of the experiments, and perspicuity in describing the mode of operation and recording the results.

Timber of all descriptions will naturally vary very considerably in the strengths of different specimens, material of organic formation being liable to an unending variety of changes in the conditions of growth: thus we observe the tensile strength of fir varies from 18,100 to 7,000 lbs. per square inch in the experiments from which the figures in Table No. 1 are collected, and other variations quite as wide are to be noticed.

The strengths in these tables are given per square inch of sectional area, taken at right angles to the strain where this latter is direct, and in inch pounds per sectional square inch for the moments of cross-breaking strain.

The strengths of the cast irons vary in tension from

5·67 tons up to as high as 10·48 tons; but the latter is exceptionally high: excluding, then, this one, the mean strength of the remaining samples is 6·54 tons, or, including all the mean, is 8·5 tons. The resistance to compression varies between 25·19 tons and 47·855 tons, showing a mean resistance of 37·76 tons for those qualities experimented upon.

From several hundred experiments made at Woolwich upon specimens of the higher qualities of cast iron, the ultimate tenacity was found to range from 10,866 lbs., or 4·85 tons, to 31,480 lbs., or 14 tons, the average being 21,173 lbs., or 9·45 tons. In 850 samples sent in for competition the tenacity ranged from 9,417 to 34,279 lbs. The ordinary cast iron of commerce gave a strength little over 6 tons, or a strength about the same as the majority in our table.

In some of Fairbairn's experiments on various kinds of cast iron, we find strengths exceeding 58 tons in compression; but this must be regarded as exceptional.

As has been already noticed, the resistance of cast-iron bars to transverse breaking is much higher than would be anticipated from the known tensile strength of the samples—a fact that has been attributed to the resistance of the layers of ferruginous material to sliding upon one another.

Tredgold's experiments on Staffordshire cast iron showed an average resistance to transverse strain of 7,645 inch lbs. for a sectional square inch.

How, then, are we to use these data when they vary so widely? is the question that arises, and it is thus answered: from the tables we may find what strength it is *reasonable to expect*, and having designed our works in accordance, we must, by specified tests, *rigorously enforced* on the manufacturer, see that we get it. In the face of the results of the experiments, as shown by the tables, it were a mere

waste of time and space to insist further upon what is so obviously necessary as the actual testing of the materials used in any important structure.

I have taken for the strengths of cast iron—tension, 7½ tons; compression, 45 tons; and shearing, 15 tons; the working strains for structures being taken as ⅓th of these, and for machinery ⅒th of these ultimate strains; for the ultimate resistance to transverse strain, 7,560 inch lbs., which corresponds to 30 cwt., on the centre of a bar 1 inch wide, 2 inches deep, and 36 inches between the bearings.

The general tensile strength of wrought-iron bars is taken at 25 tons, and plates at 22 tons; but the former, according to my own experience, is certainly too high, and probably the latter is over the general strength; in this, as with the cast iron, tests must be employed to insure the quality of the material; the strengths given in Table 8 for plates may be insisted on as reasonable for ordinary practice. The wrought iron used for machinery, being fagoted or hammered scrap, will be much stronger, running to 29 tons in tension.

Wrought iron has the great advantage of being more *reliable* than cast iron: if we know the quality of one piece, from one lot of bars or plates, we know it generally for the whole of that lot; the molecular differences which arise in different castings not occurring in the manufacture of the wrought metal. The resistance of wrought iron to transverse force may be taken at 12,000 inch lbs. It may generally be taken that the elastic power of good wrought iron is uninjured at 10 tons per sectional square inch, and for inferior iron the limit is from 8 to 10 tons. The mere loading of material once, and the constant application and removal of loads, especially if this be accompanied by vibration, are very different: hence it is important to know the effect of such vibrating forces.

In order to determine this point Mr. Fairbairn made an

experimental wrought-iron girder, and arranged testing machinery, which so acted that the load was constantly removed and replaced in such a way as to cause considerable vibration. The dimensions of the girder were as follows:—Span, 20 feet; depth, 16 inches; weight, 7 cwt. 3 qrs. The section was made up of a web plate $\frac{1}{4}$th inch thick, top flange plate 4 inches by $\frac{1}{2}$ inch thick, connected with web by a pair of angle irons, each 2 inches by 2 inches, by $\frac{5}{16}$ths inch thick, and bottom flange plate 4 inches by $\frac{1}{2}$ inch thick, connected to web by a pair of angle irons 2 inches by 2 inches, by $\frac{3}{16}$ths inch.

The beam, loaded with $\frac{1}{4}$th its calculated breaking weight, underwent continuously 596,790 changes without showing any signs of deterioration; the load was then increased to $\frac{2}{7}$ths of the breaking weight, and the beam underwent further changes to the number of 403,210, still exhibiting no weakness; the load was then increased to $\frac{2}{5}$ths the breaking weight, when, after 5,175 changes, the lower flange broke near the centre of the girder. The rivets held perfectly throughout, none of them being loosened. From this we conclude that not more than $\frac{1}{4}$th the breaking weight is safe where there is vibration in action, but that $\frac{1}{4}$th is quite safe.

In the ordinary wear of materials, it is their resistance within the limits of their elasticity that is called into action, and care must be taken that this limit is not exceeded; for when it is once impaired, the strength of the material will continue to deteriorate, and it will go on lengthening or shortening with successive loads until it ultimately breaks.

Table No. 7 shows the data of elasticity collected from various sources for materials generally, and others corresponding to the transverse resistance of timber are given in Table No. 3. These latter will be found much lower than the former.

The question of the elasticity of materials is a very

delicate one; the experiments require a microscopic care of details, and the variations give rise to much perplexity —as data from which to calculate the deflection of structures, the tabulated numbers are nearly valueless. We find the modulus of elasticity of wrought iron, for instance, vary from Hodgkinson's 24,000,000 lbs. to Rankine's 29,000,000 lbs., a difference of 20 per cent.; and this is for solid iron.

When the effects of manufacture come in, as in wrought-iron works, there is the additional element of uncertainty in the quality of the riveting. I give, however, below (for what it is worth) the modulus of elasticity determined from examples of different kinds of girders, but cannot say that in actual practice I have found these figures *fit*:—

MODULUS OF ELASTICITY FROM EXPERIMENTS ON TRANSVERSE STRAIN.

Cast-iron rectangular bars	(simple)	15,200,000 lbs.	= 6,785 tons.
„ „	(mixed)	18,892,000 „	= 8,434 „
Square and circular tubes		12,215,000 „	= 5,453 „
I Girders		13,200,000 „	= 5,893 „
Wrought-iron rolled floor beams		16,360,000 „ to 21,570,000 „	= 7,304 „ = 9,630 „
Single-webbed plate girder		14,316,000 „	= 6,391 „
Double „ „		23,610,000 „	= 10,541 „
Tubular (Conway Bridge) girder		18,754,000 „	= 8,372 „

In the rolled floor beams there is noticed a variation of nearly 32 per cent. From a number of plate-webbed girders, made from iron guaranteed at 21 tons per sectional inch, the modulus of elasticity being calculated, ranged at about 7,000 tons.

Experiments on the elasticity of materials enable us, however, to examine their molecular constitution. It is assumed that the resistances to extension and shortening are equal, and the alterations of length in simple ratio to

the load. In a number of experiments on cast iron the ratio of weight to extension, between loads of 0·47 and 6·60 tons per square inch, ranged from 117,086 to 79,576, and the apparent limit of elasticity was $\frac{1}{10}$th the breaking weight. In compression from 0·92 to 16·56 tons per square inch that ratio varied between 110,120 and 90,304, the limit of elasticity being $\frac{1}{17}$th of the weight that permanently injured the bar.

In compression at 1·84 tons, the elastic compression (recovered on removing the load) was 0·03652 inch in a 10-foot bar; the nearest load to this in the experiments on extension was 1·88 tons per square inch: $0{\cdot}03652 \times \frac{1{\cdot}88}{1{\cdot}84}$ = 0·03731 inch; the actual extension was 0·03735, or practically the same as that calculated from the compression. At 3·68 tons the compression was 0·07234; this would give for 3·76 tons per square inch $0{\cdot}07234 \times \frac{3{\cdot}76}{3{\cdot}68}$ = 0·07385 inch; that observed in extension under this load was 0·07926 inch, showing a very considerable difference. So in the compression corresponding to a load of 0·9217 ton, that calculated from the extensions is 0·0177, as against 0·0182 in the experiment; but in these instances the elasticity is imperfect, for in the extension experiments permanent set occurred at 0·7 ton, and in compression at 0·92 ton per square inch.

As to the uniformity of extension, all we have practically to deal with are those up to the highest strain used in practice—say up to 3 tons per sectional square inch—which is used in some hydraulic apparatus. Taking the first experimental number, the extensions corresponding to the increasing weights are calculated by the common formula for comparison with the actual extensions. In compression we take up to 9 tons in the same way; the bars were 10 feet long.

Load—Tons per Sq. Inch.	Extensions in Inches.	Calculated Extensions.	Load—Tons per Inch.	Compressions in Inches.	Calculated Compressions.
0·47	0·00900	—	0·9217	0·01828	—
0·70	0·01348	0·01340	1·8400	0·03652	0·03648
0·94	0·01805	0·01800	2·7600	0·05578	0·05473
1·41	0·02763	0·02700	3·6800	0·07234	0·07297
1·88	0·03735	0·03600	4·6000	0·09097	0·09122
2·35	0·04735	0·04500	5·5200	0·10942	0·10946
2·82	0·05758	0·05400	6·4400	0·12758	0·12770
			7·3600	0·14626	0·14595
			8·2800	0·16454	0·16419
			9·2100	0·18140	0·18263

From these tables the compressions follow more closely the generally received law than the extensions, and they may be regarded as fairly supporting the theory. The following results are found from experiments on wrought iron:—

In 20 readings under loads from 0·56 tons to 11·26 tons per sectional square inch, the ratio of the weight to the extension varies from 219,459 to 242,665, the next highest being 234,982, so that the first is probably exceptional; the mean is 230,760. Here, then, the extensions are tolerably regular.

In the experiments on the cast-iron bars, the rods in compression were cased to prevent deflection. The modulus of elasticity will be seen to have varied in extension from 14,050,320 lbs. to 12,377,040 lbs., the mean of these two being 13,213,680 lbs.; in compression from 13,214,400 lbs. to 12,013,680, of which the mean is 12,614,040 lbs.

The modulus of elasticity of wrought iron, corresponding to the mean ratio of weight to extension, as given above, is 27,691,200 lbs.

TABLE No. 1.

ULTIMATE TENSILE RESISTANCE OF TIMBER IN LBS. PER SQUARE INCH OF SECTION.

Ash	19,600 to 15,784
Beech	22,200 „ 11,500
Elm	14,400 „ 13,489
Fir	18,100 „ 7,000
„ American	12,000
„ Memel	11,000
„ Riga	12,600
„ Mar Forest	12,000
Larch	10,200
Hornbeam	20,240 to 4,253
Mahogany	21,800 „ 8,000
Oak	19,800 „ 9,000
„ English	15,000
„ African	14,400
„ Canadian	12,000
„ Dantzic	14,500
Teak	15,000 to 8,200

TABLE No. 2.

ULTIMATE RESISTANCE OF TIMBER TO CRUSHING IN LBS. PER SQUARE INCH OF SECTION.

Ash	9,363 to 8,683
Beech	9,363 „ 7,733
Elm	10,331
Fir	6,819 „ 5,375
Hornbeam	7,289 „ 4,533
Mahogany	8,280
Oak, English	10,058 „ 6,484
„ Dantzic	7,731
Teak	12,101

TABLE No. 3.

Transverse Resistance of Timber.

E, modulus of elasticity; S, ultimate moment of resistance in inch lbs.

	E.	S.
Teak	603,600	2,462
Poon	422,400	2,221
English Oak	218,400	1,181
" "	362,800	1,672
Canada "	536,200	1,766
Dantzic "	297,800	1,457
Adriatic "	243,600	1,383
Ash	411,200	2,026
Beech	338,400	1,556
Elm	174,960	1,013
Pitch Pine	306,400	1,632
Red "	460,000	1,341
New England Fir	547,800	1,102
Riga "	332,200	1,108
Mar Forest "	161,340	1,144
Larch	154,080	853
Norway Spar	364,400	1,474

TABLE No. 4.

Ultimate Tensile Resistance of Metals in Tons per Square Inch of Section.

Cast Iron, cast horizontally	8·48
" " vertically	8·70
Tilted Cast Steel	59·93
Hammered Blister Steel	59·43
Sheer Steel	56·97
Welsh Wrought Iron	29·30
Stafford " "	27·15
Swedish " "	29·00
Fagoted " "	29·00
Iron Wire	38·40
Hard Gun Metal	16·23
Hammered Wrought Copper	15·08
Cast Copper	8·51
Fine Yellow Brass	8·01
Cast Tin	2·11
Cast Lead	0·81

TABLE No. 5.

ULTIMATE RESISTANCE OF VARIOUS CAST IRONS IN TONS PER SQUARE INCH OF SECTION.

T, resistance to tensile force.
C, „ crushing „

	T.	C.
Low Moor, No. 1	5·667	28·809 25·198
„ No. 2	6·201	44·430 41·219
Clyde, No. 1	7·198	41·459 39·616
„ No. 2	7·949	49·103 45·549
„ No. 3	10·477	47·855 46·821
Blaenavon, No. 1	6·222	40·562 35·964
„ No. 2	6·380	30·606 30·594
Calder, No. 1	6·131	32·229 33·921
Coltness, No. 3	6·820	44·723 45·460
Brymbo, No. 1	6·440	33·390 33·784
„ No. 3	6·923	33·988 34·356
Bowling, No. 2	6·032	33·987 33·028

TABLE No. 6.

ULTIMATE RESISTANCE OF VARIOUS BUILDING MATERIALS TO CRUSHING FORCE IN LBS. PER SQUARE INCH OF SECTION.

Portland Stone	1,284
Statuary Marble	3,216
Craigleith „	8,688
Chalk	501
Pale Red Brick	562
Gloucester Roe Stone	644
Red Brick	808
Fire Brick	1,717

Derby Grit	3,142
Killaly White Freestone	4,566
Portland „	4,571
Craigleith „	5,487
York Paving	5,714
White Statuary Marble	6,059
Bramley Fall Sandstone	6,059
Cornish Granite	6,364
Dundee Sandstone	6,630
Craigleith „	6,916
Devon Red Marble	7,428
Compact Limestone	7,713
Peterhead Granite	8,283
Black Compact Limestone	8,855
Purbeck	9,160
Black Brabant	9,219
Blue Aberdeen Granite	10,914

TABLE No 7.

MODULUS OF ELASTICITY AND LIMIT OF ELASTIC RESISTANCE OF VARIOUS MATERIALS IN LBS. PER SQUARE INCH OF SECTION.

Material.	Modulus.	Limits.
Brass	8,930,000	6,700
Gun Metal	9,873,000	10,000
Cast Iron	18,400,000	15,300
Wrought Iron	24,920,000	17,800
Lead	720,000	1,500
Steel	29,000,000	45,000
Tin	4,608,000	2,880
Zinc	13,680,000	5,700
Marble	2,520,000	4,900
Slate	15,800,000	—
Portland Stone	1,533,000	1,500
Ash	1,640,000	3,540
Beech	1,345,000	2,360
Elm	1,340,000	3,240
Fir (Red)	2,016,000	4,290
Larch	1,074,000	2,360
Mahogany	1,596,000	3,800
Oak	1,700,000	3,960

TABLE No. 8.

SUMMARY OF ULTIMATE AND WORKING RESISTANCES OF VARIOUS MATERIALS IN TONS PER SQUARE INCH OF SECTION.

Material.	Ultimate Strength.			Working Strength.		
	Tension	Compression.	Shearing.	Tension.	Compression.	Shearing.
Steel Bars	45	70	30	9	9	6
„ Plates	40	—	—	8	—	—
Wrought-iron Bars	25	17	22	5	$3\frac{1}{2}$	$4\frac{1}{2}$
„ Plates	22	17	20	$4\frac{1}{2}$	$3\frac{1}{2}$	4
Iron Wire Cable	40	—	—	8	—	—
Cast Iron	$7\frac{1}{2}$	45	15	$1\frac{1}{2}$	9	3
Ash	$7\frac{1}{2}$	4	$0\frac{3}{4}$	$1\frac{1}{4}$	$0\frac{2}{3}$	$0\frac{1}{6}$
Beech	5	4	—	1	$0\frac{2}{3}$	—
Elm	6	$4\frac{1}{2}$	$0\frac{3}{4}$	1	$0\frac{2}{3}$	$0\frac{1}{8}$
Fir	5	$2\frac{1}{2}$	$0\frac{1}{2}$	1	$0\frac{1}{2}$	$0\frac{1}{12}$
Oak	$6\frac{1}{2}$	$3\frac{1}{2}$	1	1	$0\frac{1}{2}$	$0\frac{1}{6}$
Teak	$6\frac{1}{2}$	5	—	1	1	—
Granite	—	$3\frac{1}{2}$	—	—	$0\frac{1}{2}$	—
Sandstone	—	$1\frac{1}{2}$	—	—	$0\frac{1}{4}$	—

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